NOTES AND CORRESPONDENCE

Effects of Variations in Precipitation Size Distribution and Fallspeed Law Parameters on Relations between Mean Doppler Fallspeed and Reflectivity Factor

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ABSTRACT

A description is given of the influence of variations in the form of the hydrometeor particle size distribution and of variations in the form of the particle fallspeed law on relations between mean Doppler fallspeed v_D and reflectivity factor Z. It is shown that for rain, variations in distribution shape can produce errors in v_D of 20%–25%. The resultant errors in raindrop distribution parameters deduced from v_D and Z are calculated. Uncertainties regarding hydrometeor phase (and thus in the particle fallspeed law) are shown to produce much larger errors in the value of v_D deduced from Z of up to a factor of 4. These results are deduced theoretically, tested with experimental raindrop size spectra, and demonstrated with experimental Doppler radar data for a tropical thunderstorm. It is concluded that the use of theoretical or empirical v_D–Z relations to deduce particle size distributions will produce large errors in most meteorological situations. However, in those cases where a reasonable assumption can be made about particle phase, the use of such relations may produce estimates of vertical winds with acceptable accuracy.

1. Introduction

The purpose of this note is to demonstrate the effects of variations in precipitation size distribution and particle fallspeed law parameters on the relationship between mean Doppler fallspeed v_D and radar reflectivity factor Z. Such v_D–Z relations are used commonly in radar meteorology to correct measurements of mean Doppler velocity for the presence of vertical winds, thereby enabling a determination of the particle size distribution from the Doppler spectrum when the measurements are made at vertical incidence. Atlas et al. (1973) have reviewed various empirical and theoretical relations of this type for rain and have shown that deviations from these relations can result in very large errors in the raindrop size distribution. In an effort to reduce these errors, Srivastava (1990) has devised a method that combines vertically pointing Doppler radar data with VAD measurements, which would be especially useful in stratiform rain where the rainfall is relatively horizontally uniform. Such a procedure, however, would probably be subject to large errors when used in a thunderstorm.

It will be shown here that deviations of the type considered by Atlas et al. (1973) can be caused by moment-to-moment variations in the shape or form of the size distribution. It will also be shown that uncertainties in the phase or type of the hydrometeors (and therefore in the particle fallspeed law) can produce very large deviations from an assumed v_D–Z law. These results are shown theoretically and demonstrated empirically using experimental disdrometer data, and are further elucidated using Doppler radar data at vertical incidence for a tropical thunderstorm. The implications of this work are that such v_D–Z relations are of limited usefulness in determining the particle size distribution from Doppler radar data for storms unless a priori knowledge is available concerning the shape of the distribution and the form of the fallspeed law. However, if an assumption can be made about the phase of the particles, the use of v_D–Z relations may produce estimates of vertical winds with acceptable errors. Finally, Steiner (1991) has suggested a method for estimating vertical winds and drop size distributions that uses a relation between the mean Doppler fallspeed and the differential reflectivity. A discussion is given of how his method could be combined with the results presented here.

2. Theoretical relations

The radar integral parameters of interest in this work are the mean Doppler fallspeed of precipitation v_D and the radar reflectivity factor Z. The former is defined by

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$$v_D = \frac{\int_{D_{\min}}^{D_{\max}} v(D)D^6N(D)dD}{\int_{D_{\min}}^{D_{\max}} D^6N(D)dD}, \quad (1)$$

where it has been assumed that the precipitation particles can be described by a sphere with equivalent spherical diameter D, that the fallspeeds in still air of these particles are $v(D)$, and that they are distributed with respect to size according to the distribution function $N(D)$ between minimum and maximum sizes $D_{\min}$ and $D_{\max}$, respectively. The reflectivity factor Z is given by

$$Z = 10^6 \int_{D_{\min}}^{D_{\max}} D^6N(D)dD. \quad (2)$$

In these definitions it has been assumed that the backscattering cross sections of the particles can be approximated by Rayleigh cross sections. It will be further assumed in this work that $D_{\min} = 0$, $D_{\max} \rightarrow \infty$, and that the distribution can be approximated by a gamma distribution of the form

$$N(D) = N_0D^{\mu}\exp(-\Lambda D), \quad (3)$$

where N_0, μ, and Λ are parameters of the distribution. In these equations the following units for these quantities are assumed: D (cm), v_D (m s^{-1}), Z (mm^6 m^{-3}), $N(D)$ (m^{-3} cm^{-1}), N_0 (m^{-3} cm$^{-1-\mu}$), and Λ (cm^{-1}). With this form for the size distribution the reflectivity factor becomes

$$Z = \frac{N_0 10^6\Gamma(7+\mu)D_0^{7+\mu}}{\alpha^{7+\mu}}, \quad (4)$$

where D_0 (cm) is the median volume diameter, $\Gamma(x)$ is the complete gamma function, and α is the relationship between D_0 and Λ as found by Ulbrich (1983); that is,

$$\alpha = \Lambda D_0 = 3.67 + \mu + 10^{-0.3(9+\mu)}. \quad (5)$$

To explore the relationship between v_D and Z, it is necessary to assume a dependence of the fallspeed $v(D)$ on D. A commonly used form for raindrops, which is a close approximation to the actual fallspeeds measured by Gunn and Kinzer (1949), is the empirical expression deduced by Atlas et al. (1973); namely,

$$v(D) = 9.65 - 10.3\exp(-6D). \quad (6)$$

If Eq. (6) is substituted into Eq. (1) then it follows that

$$v_D = 9.65 - 10.3\left(\frac{\Lambda}{6+\Lambda}\right)^{7+\mu}$$
$$= 9.65 - 10.3\left(1 + \frac{6D_0}{\alpha}\right)^{-(7+\mu)}, \quad (7)$$

so that elimination of D_0 between Eqs. (4) and (7) defines a relationship between v_D and Z for given values of N_0 and μ; that is,

$$v_D = 9.65 - 10.3 \times \left\{1 + 6\left[\frac{Z}{N_0 10^6\Gamma(7+\mu)}\right]^{1/(7+\mu)}\right\}^{-(7+\mu)}. \quad (8)$$

In using Eq. (6) in Eq. (1), it has been assumed that $D_{\min} = 0$, whereas the range of validity of this equation is $D_{\min} \geqslant 0.01$ cm. It is easy to show from Eqs. (1) and (6) that the error that results from assuming that $D_{\min} = 0$ in Eq. (1) is negligible for all values of μ as long as $D_0 \geqslant 0.015$ cm. Eq. (8) is plotted in Fig. 1 for values of μ ranging from -1 to 3, the range of values within which lie almost all of the empirical results (Ulbrich 1983). Each of the v_D–Z curves in Fig. 1 is labeled with the value of μ to which it corresponds. In calculating these curves the empirical relationship between N_0 and μ found by Ulbrich (1983) has been employed; namely,

$$N_0 = 6.4 \times 10^4 e^{3.2\mu}. \quad (9)$$

However, deviations of a factor of ten either side of this expression have been considered and the maximum deviations are shown as the heavy curves above and below the calculated results. In other words, the upper curve is for $\mu = -1$ and $N_0 = 6.4 \times 10^3 e^{3.2\mu}$ and the lower curve is for $\mu = 3$ and $N_0 = 6.4 \times 10^5 e^{3.2\mu}$. This range of values for N_0 is shown by Ulbrich (1983) to encompass all of the results deduced from empirical

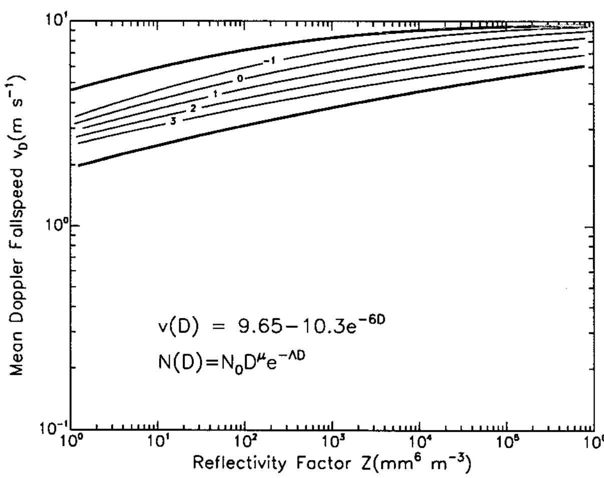

FIG. 1. The dependence of the mean Doppler fallspeed on radar reflectivity factor for raindrops. It has been assumed that the size distribution can be approximated by a gamma distribution and that the coefficient N_0 is related to μ by the empirical relation given by Ulbrich (1983). The heavy solid curves depict the maximum deviations from the theoretical expressions due to variations in N_0 of a factor of 10 from the latter relation. Each of the curves is labeled with the value of the exponent μ to which it corresponds.

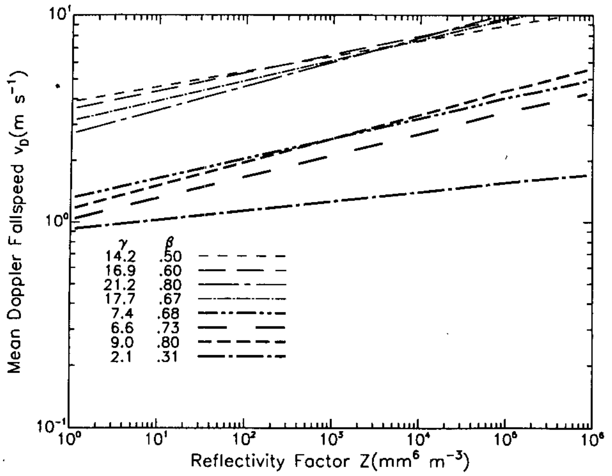

FIG. 2. The dependence of mean Doppler fallspeed on reflectivity factor for precipitation particles having an exponential size distribution and fallspeeds described by a power-law relation of the form of Eq. (9). The eight curves shown in the figure correspond to the various fallspeed laws and types of precipitation particles listed in Table 1.

relations between remote measureables and rainfall rate. It is seen from these results that small variations in the shape of the distribution (i.e., μ) will result in deviations in v_D and thus in the size distribution parameters deduced from it. In the next section these deviations in distribution shape will be demonstrated using experimental disdrometer and drop camera data. The errors in distribution parameters deduced from v_D and Z that result from the deviations described above are calculated in the appendix.

Much larger deviations will result from uncertainties associated with the fallspeed law of the precipitation particles, especially if the phase of the particles is uncertain. Although Eq. (6) is an accurate representation of the fallspeeds of raindrops, it cannot be used for particles of different phase, such as, graupel, hail, and snow. For these particles experimental work has shown that the fallspeeds are best approximated by a power law of the form

$$v(D) = \gamma D^{\beta}. \qquad (10)$$

Typical values of the constants γ and β are shown in Table 1 for four fallspeed laws that apply to particles having phase different from that of raindrops, namely, three that apply to graupel particles, and one which is for snow. Also shown for comparison are values of γ and β for four fallspeed laws proposed for raindrops. With the form of the fallspeed law given by Eq. (10), substitution into Eq. (1) yields

$$v_D = \frac{\gamma\Gamma(7 + \mu + \beta)D_0^{\beta}}{\Gamma(7 + \mu)\alpha^{\beta}}. \qquad (11)$$

Elimination of D_0 between Eqs. (4) and (11) produces a v_D–Z expression of the form

$$v_D = \frac{\gamma\Gamma(7 + \mu + \beta)}{(10^6 N_0)^{\beta/(7+\mu)}[\Gamma(7 + \mu)]^{(7+\mu+\beta)/(7+\mu)}} Z^{\beta/(7+\mu)}. \qquad (12)$$

It has been assumed here that Z is the equivalent reflectivity factor of the scatterers. It follows that the dependence on the density of the material of which the particles are composed is contained in γ and on the dielectric constant is contained in Z. For given values of γ, β, N_0, and μ, this equation represents a power-law relation between v_D and Z similar to the theoretical relation of Rogers (1964) and the empirical results of Joss and Waldvogel (1970). Taking $N_0 = 8 \times 10^4$ m^{-3} cm^{-1} and $\mu = 0$ [the Marshall and Palmer (1948) values for raindrops], the eight relations corresponding to the pairs of values of γ and β are plotted in Fig. 2. The value of $\mu = 0$ (exponential distribution) is a reasonable assumption for each of the types of particles in Table 1, but the assumed value of N_0 may not be accurate. Nevertheless, the exponent on N_0 in Eq. (12) is small enough that v_D is not as sensitive to variations in N_0 as to those in γ and β.

The four relations that apply to rain show little dependence of the v_D–Z relation on the form of the fallspeed law, especially when compared to the dependence on variations in particle size distribution shape in Fig. 1. However, there is a very strong dependence on γ and β in the case where the particles in the beam are graupel or snow rather than rain. In fact, there is a difference of up to a factor of four between the relations that apply to rain and those for graupel. The relation for snow deviates even further from those for rain. It may thus be concluded from these theoretical results that the use of v_D–Z relationships to deduce vertical winds and particle size distributions will yield valid results only in the special case where the phase of the hydrometeors is known and where there are minimal variations in particle distribution shape from moment

TABLE 1. Coefficients γ and exponents β in power-law approximations to the fallspeeds of hydrometeors of the form $v(D) = \gamma D^{\beta}$. The units of γ are such that when D is in centimeters, $v(D)$ is in meters per second. Also shown are the source from which these data were taken and the type of hydrometeor.

γ	β	Source	Particle type
14.2	0.5	Spilhaus (1948)	raindrops
16.9	0.6	Sekhon and Srivastava (1971)	raindrops
21.15	0.8	Liu and Orville (1968)	raindrops
17.67	0.67	Atlas and Ulbrich (1977)	raindrops
7.44	0.675	Locatelli and Hobbs (1974)	lump graupel
6.64	0.725	Locatelli and Hobbs (1974)	soft lump graupel
9.0	0.8	Auer (1972)	large graupel
2.07	0.31	Langleben (1954)	snow

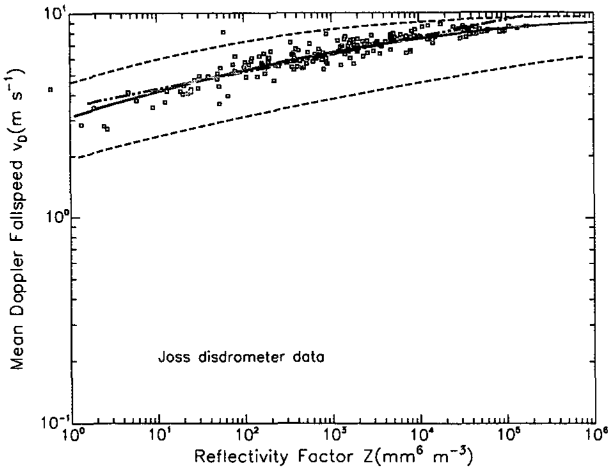

FIG. 3. Plot of v_D versus Z values as determined from experimental raindrop size spectra collected with a Joss disdrometer. The heavy solid curve is the same as in Fig. 1 for $\mu = 0$. The heavy dash-dot line is a power-law fit to the data of the form $v_D = 3.5Z^{0.084}$. The dashed curves are the same as those in Fig. 1 depicting the maximum variations in the distribution parameter N_0.

to moment. Such a case might occur in relatively quiet meteorological conditions (such as, stratiform rain), but in the turbulent environment within a thunderstorm, large errors should be expected from the use of a v_D–Z relation. The errors in distribution parameters deduced from v_D and Z when the particle fallspeed is given by Eq. (9) are described in the appendix.

3. Experimental tests

As tests of the conclusions in the previous section, two sets of raindrop size spectra have been used to simulate the experimental technique where rain is observed at vertical incidence by a Doppler radar. The first set of raindrop size spectra were collected with a Joss disdrometer and a description of these data is given in Atlas and Ulbrich (1977). A complete description of the instrumentation used to acquire the data is given in Joss and Waldvogel (1967). The second set of data consists of drop camera measurements acquired by the Illinois State Water Survey at the University of Illinois. Although the results shown in this work are for a limited subset of these data, they are typical of those found for the remainder of the dataset. A complete description of the dataset and the methods used to acquire the data is given in Mueller (1965).

For each dataset the measurements consist of numbers of raindrops per unit volume $n(D_i)$ (m^{-3}) for particles of diameter D_i (cm) in size categories of width ΔD_i (cm). In terms of these definitions the relevant quantities of interest in this work are calculated from

$$Z = 10^6 \sum_{i=1}^{N} D_i^6 n(D_i) \qquad (13)$$

for the reflectivity factor Z and

$$v_D = \frac{\sum_{i=1}^{N} v(D_i) D_i^6 n(D_i)}{\sum_{i=1}^{N} D_i^6 n(D_i)} \qquad (14)$$

for the mean Doppler fallspeed v_D, where $v(D_i)$ is the Gunn and Kinzer (1949) fallspeed of a particle of diameter D_i. The results for the set of measurements collected with a disdrometer are shown in Fig. 3. Also shown in this figure is the theoretical relation corresponding to $\mu = 1$, which fits the data quite well, and a power-law fit of the form of Eq. (12); that is, $v_D = aZ^b$ with $a = 3.5$ and $b = 0.084$. (The units of the coefficient a are such that when Z is in its standard units (mm^6 m^{-3}), v_D is in meters per second.) Similar results are shown in Fig. 4 for the set of data taken with a drop camera. As in Fig. 3 the solid curve is the theoretical result for $\mu = 1$. For both of these sets of data there are significant deviations of individual data points from the theoretical curve. In Fig. 3 these amount to about 18%, whereas in Fig. 4 they are about 25%. Virtually all of the data in Figs. 3 and 4 lie within the bounds defined by the heavy solid curves in Fig. 1 corresponding to the maximum variations in the distribution parameter N_0. The latter curves are reproduced in Figs. 3 and 4 as the dashed curves for reference. These simulations show that moment to moment variations in the shape of the raindrop size distribution can produce deviations from a theoretical or empirical v_D–Z law that may produce useful estimates of vertical winds with acceptable errors but which will yield estimates of drop size distributions that contain very large errors.

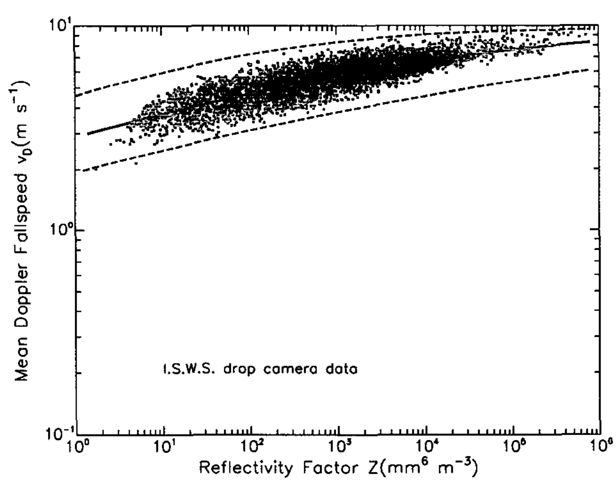

FIG. 4. Plot of v_D versus Z values as determined from experimental raindrop size spectra collected with raindrop camera. The heavy solid curve is the same as in Fig. 1 for $\mu = 1$. The dashed curves are the same as in Fig. 3.

Additional confirmation of these conclusions is contained in a set of Doppler radar measurements made at vertical incidence in a tropical thunderstorm in Arecibo, Puerto Rico. A complete description of these data and the equipment used to acquire them is contained in Chilson et al. (1993). The essential features of these measurements are that they were made with dual-wavelength (UHF and VHF) radars and involve wavelengths that are sufficiently long that both the clear air and precipitation parts of the Doppler spectrum could be discerned. This therefore enabled a direct and simultaneous measurement of the vertical winds and the contribution of the precipitation to the Doppler spectrum. The results so obtained are plotted in Fig. 5 together with the v_D–Z relations shown in Fig. 2. All of the data in Fig. 5 involve radar measurements taken above the freezing level in the turbulent environment within a thunderstorm, so they probably involve hydrometeors with a variety of phases, including supercooled liquid water drops, frozen graupel, and possibly snow. It is therefore not surprising that the data would show very large scatter. However, almost all the data lie within the bounds defined by the power-law fallspeed laws given in Table 1, which indicates that there are no unreasonable results for v_D and Z deduced by the methods employed in the analysis of the Doppler radar data. The results further confirm that there is no single v_D–Z relation that can be employed in the analysis of Doppler radar data in a thunderstorm.

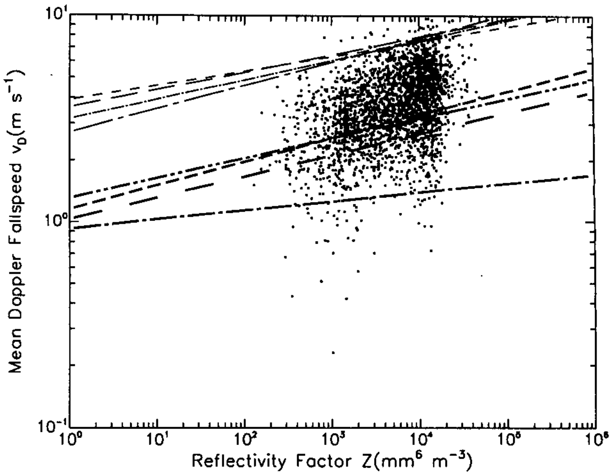

FIG. 5. Plot of v_D versus Z values as determined from dual-wavelength Doppler radar measurements made at vertical incidence in a tropical thunderstorm. The eight straight lines in the figure are the same as those in Fig. 2 corresponding to v_D–Z relations found using the power-law approximation to the particle fallspeeds.

4. Conclusions

This work has explored the effects of variations of parameters associated with hydrometeor size distributions and particle fallspeed laws on the relationship between the mean Doppler fallspeed and radar reflectivity factor. It has been shown that for raindrops the important variations are those associated with drop size distribution shape and that these produce maximum variations in v_D of about 20%–25% from an assumed theoretical or empirical v_D–Z relation. These variations can result in large errors in estimates of drop size distribution parameters but may produce estimates of vertical winds with acceptable errors if a valid assumption can be made about the phase of the particles. It has also been shown that in situations where the hydrometeor phase is uncertain, the assumption of a particular phase can produce very large errors in v_D (a factor of 4 or more). The results for raindrops have been verified using experimental drop size spectra collected with a disdrometer and with a drop camera in a simulation of Doppler radar measurements at vertical incidence. The results concerning hydrometeor phase have been illustrated using v_D–Z results deduced from actual dual-wavelength Doppler radar measurements in a thunderstorm. These results lead to the conclusion that the use of theoretical or empirical v_D–Z relations to find vertical winds and particle size distributions is a valid process only in special situations, namely, when the shape of the size distribution and the phase of the hydrometeors are known a priori. This may be the case in stratiform rain below the bright band but may not be true in thunderstorms above the freezing level.

A method for reducing the uncertainties involved in observations of the type described here has been suggested by Steiner (1991). He has shown that for rainfall rates $R \geqslant 1$ mm h^{-1} an accurate relationship exists between the mean Doppler fallspeed and differential reflectivity Z_{DR} of raindrops. This implies a multiparameter measurement technique wherein two radars would be used, the first a Doppler radar to observe the mean Doppler velocity of the precipitation particles at vertical incidence, the second a multiparameter or polarization radar to observe in a horizontally scanning mode the differential reflectivity and other parameters, such as, the linear depolarization ratio and the specific differential phase shift. The differential reflectivity would then be used to determine the mean Doppler fallspeed v_D from the v_D–Z_{DR} relation proposed by Steiner and this is then used with the vertical incidence measurements to find the vertical wind and the drop size distribution. An analysis of the errors involved in determining the distribution has not been performed by Steiner, but he does point to the difficulties associated with attempting to acquire accurate data with two radars, which would necessarily have mismatched sampling volumes since the Doppler radar would be directly beneath the precipitation and the polarization radar would be located some horizontal distance remote from the first. Nevertheless, the use of a polar-

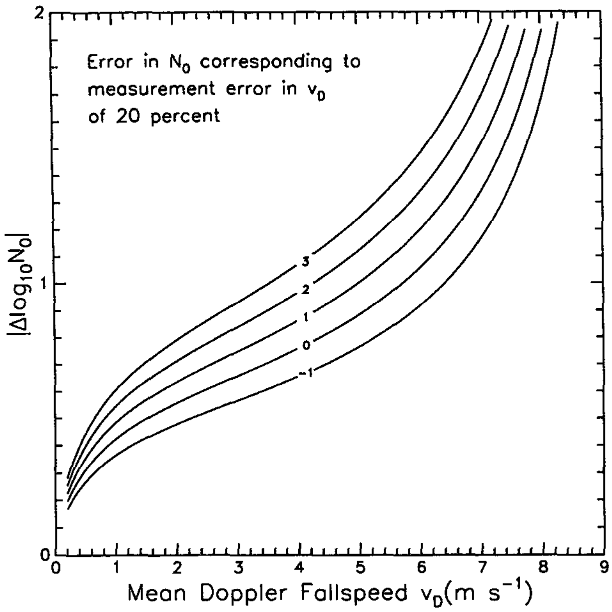

FIG. A1. Error in the distribution parameter N_0 for raindrops corresponding to a measurement error in v_D of 20% as a function of v_D with Z held constant. Each of the curves is labeled with the value of μ to which it corresponds. The fallspeeds of the particles are assumed to be given by Eq. (6) for raindrops.

ization radar in this way would permit discrimination between regions composed entirely of raindrops and those of different phase, thus eliminating one of the principal sources of error in estimating vertical winds as discussed in this work.

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APPENDIX

Error Analysis

In this appendix the errors in particle size distribution parameters deduced from v_D and Z that result from variations in v_D are described. These can be determined for rain using Eqs. (4) and (7). If D_0 is eliminated between these two expressions, the result can be solved for N_0 to give

$$N_0 = \frac{Z 6^{7+\mu}}{10^6 \Gamma(7+\mu)} \left[\left(\frac{a_1 - v_D}{a_2} \right)^{-1/(7+\mu)} - 1 \right]^{-(7+\mu)}, \tag{A1}$$

where $a_1 = 9.65$ m s^{-1} and $a_2 = 10.65$ m s^{-1}. From this expression it may be shown that for given Z and μ, the change in the magnitude of the common logarithm of N_0 due to variations in v_D is given by

$$|\Delta \log_{10} N_0| = \frac{v_D}{2.3 a_2} \left(\frac{a_1 - v_D}{a_2} \right)^{-(8+\mu)/(7+\mu)} \times \left[\left(\frac{a_1 - v_D}{a_2} \right)^{-1/(7+\mu)} - 1 \right]^{-1} \left(\frac{\Delta v_D}{v_D} \right). \tag{A2}$$

This expression is plotted in Fig. A1 for values of μ ranging from -1 to 3 and for $\Delta v_D / v_D = 20\%$. The figure depicts the errors that can be expected in N_0 due solely to variations in v_D when the reflectivity factor Z and the distribution shape μ are constant. Figures for other values of $\Delta v_D / v_D$ are similar to Fig. A1 with the curves displaced upward for larger values of $\Delta v_D / v_D$. It is clear from this figure that for variations in v_D of the magnitude found in the computer simulations, errors in N_0 of at least an order of magnitude could result.

Similar procedures can be followed to find expressions describing the errors in D_0 and μ due to measurement errors in v_D with Z held constant. The results are

$$\frac{\Delta D_0}{D_0} = \frac{v_D[(a_1 - v_D)/a_2]^{-(8+\mu)/(7+\mu)}}{\{[(a_1 - v_D)/a_2]^{-1/(7+\mu)} - 1\}} \left(\frac{\Delta v_D}{v_D} \right) \tag{A3}$$

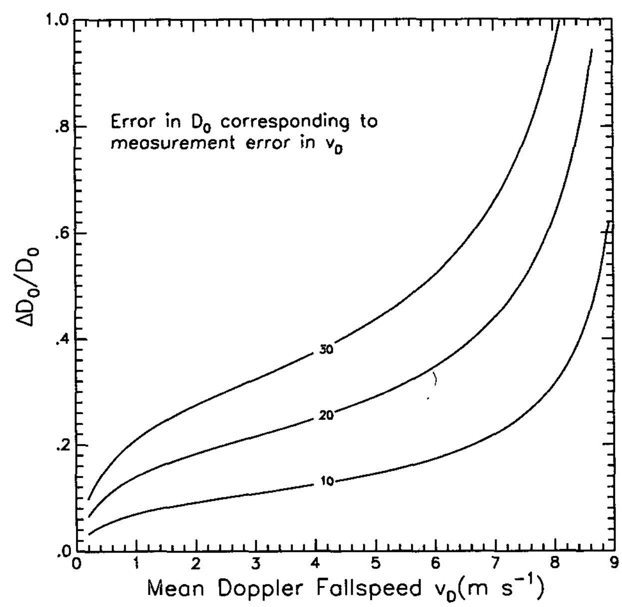

FIG. A2. Fractional error in D_0 for raindrops corresponding to measurement errors in v_D of 10%, 20%, and 30% with Z held constant. The curves are labeled with the value of the measurement error in v_D. Each curve corresponds to a value of $\mu = 0$ but is virtually identical to those for other values of μ. The fallspeeds of the particles are assumed to be given by Eq. (6) for raindrops.

and

$$|\Delta\mu| = \frac{\alpha v_D[(a_1 - v_D)/a_2]^{-(8+\mu)/(7+\mu)}}{a_2(7+\mu)\{[(a_1 - v_D)/a_2]^{-1/(7+\mu)} - 1\}} \times \left(\frac{\Delta v_D}{v_D}\right), \quad \text{(A4)}$$

Eqs. (A2) and (A3) are plotted in Figs. A2 and A3, respectively. The curves in Fig. A2 are labeled with the measurement error in v_D with values of 10%, 20%, and 30% and each of these curves has been calculated for a value of $\mu = 0$. However, curves for other values of μ are virtually identical to those in Fig. A2. The curves in Fig. A3 are labeled with the value of μ to which they correspond.

The errors in distribution parameters corresponding to errors in v_D when the particle fallspeeds are approximated by a power law of the form given by Eq. (10) can be shown to be

$$|\Delta \log_{10} N_0| = \frac{(7+\mu)}{2.3\beta}\frac{\Delta v_D}{v_D} \quad \text{(A5)}$$

and

$$\frac{\Delta D_0}{D_0} = \frac{1}{\beta}\frac{\Delta v_D}{v_D}. \quad \text{(A6)}$$

A simple expression of this type for the error in μ is not possible, but a plot of $|\Delta\mu|$ versus μ for measurement errors in v_D of 10%, 20%, and 30% is shown in Fig. A4. These results are similar in magnitude to those found from Eqs. (A1)–(A3) but, for given $\Delta v_D/v_D$ the errors in N_0 and D_0 are independent of v_D. As an example, for $\mu = 0$, $\beta = 0.67$, and $\Delta v_D/v_D = 0.2$, it is found from Eqs. (A5) and (A6) that $|\Delta \log_{10} N_0| = 0.91$ and $\Delta D_0/D_0 = 0.3$, whereas from Fig. A4, $|\Delta\mu| = 1.5$. These values are similar to those displayed in Figs. A1, A2, and A3, respectively.

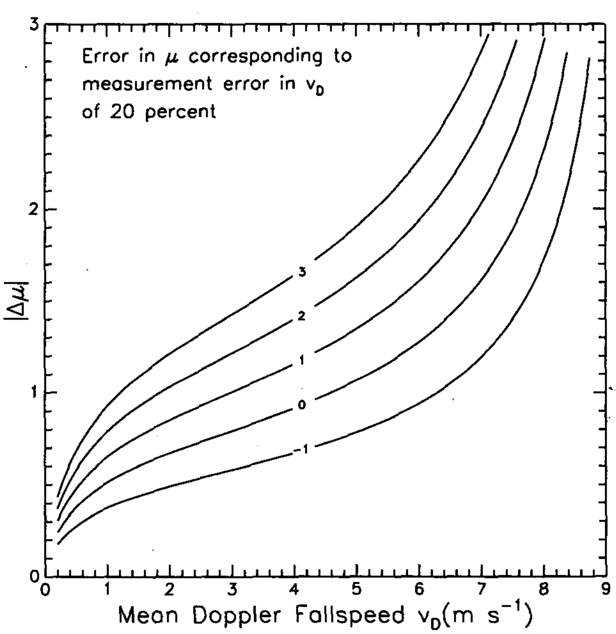

FIG. A3. Error in μ for raindrops corresponding to a measurement error in v_D of 20% with Z held constant. Each of the curves is labeled with the value of μ to which it corresponds. The fallspeeds of the particles are assumed to be given by Eq. (6) for raindrops.

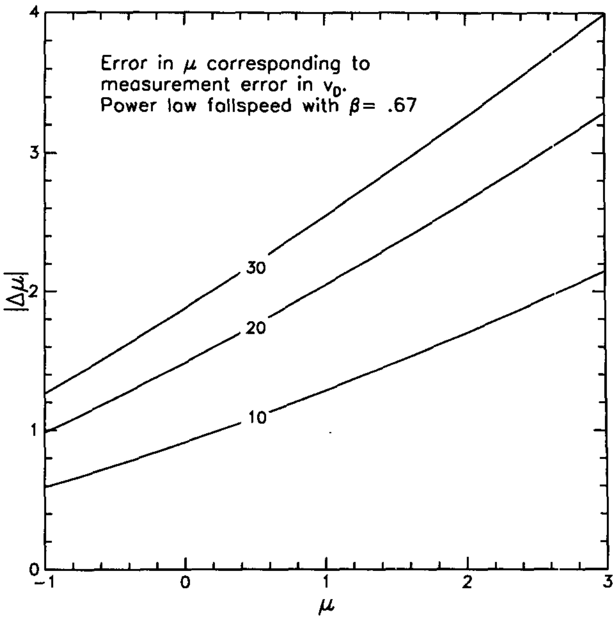

FIG. A4. Error in μ corresponding to measurement errors in v_D of 10%, 20%, and 30%. Each of the curves is labeled with the value of $\Delta v_D/v_D$ in percent to which it corresponds. The fallspeeds of the particles are assumed to obey the power-law relation given by Eq. (10).

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