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Performance of correlation estimators for spaced-antenna wind measurement in the presence of noise

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[1] The theoretical accuracy of baseline winds, estimated using spaced antennas (SA) and a full correlation analysis (FCA) method to process signals in the presence of noise, is derived assuming horizontally isotropic refractive index perturbations with a horizontal scale small compared to the transmitting antenna diameter *D*. This performance of the FCA method is compared with the theoretical performance of another correlation-based approach (i.e., the cross-correlation ratio method, CCR). The theoretical results of the error analysis are supported with numerical simulations and experimental data. It is shown that the theoretical analysis is valid and the results can be applied to improve wind estimates obtained from SA signals contaminated with additive white noise. The theory shows that the effect noise has on SA wind estimates depends on system configuration and lag spacing, and cannot be correctly accounted by a reduced correlation coefficient due to noise as hypothesized by *May* [1988]. *INDEX TERMS:* 6952 Radio Science: Radar atmospheric physics; 6969 Radio Science: Remote sensing; 6974 Radio Science: Signal processing; 6994 Radio Science: Instruments and techniques; *KEYWORDS:* spaced antenna technique, wind measurement cross-correlation ratio (CCR), full correlation analysis (FCA), signal-to-noise ratio (SNR)

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1. Introduction

[2] The SA technique measures both the radial and cross-beam wind components within the radar's resolution volume, and thus potentially provides wind data with higher resolution than Doppler beam swinging (DBS) methods (i.e., a variant of the velocity azimuth display, VAD, technique first proposed by *Lhermitte and Atlas* [1961]). The DBS method is today the most widely used wind profiling method [*Strauch et al.*, 1984]. The theory of the SA technique developed by *Briggs et al.* [1950] is based on the motion of the interference pattern formed from waves scattered by random perturbations in the propagation medium (i.e., due to variations in electron density, refractive index perturbations, etc.). The most commonly used data analysis method for the SA technique is Briggs' full correlation analysis (FCA) [*Briggs et al.*, 1950; *Briggs*, 1984], in which the wind is estimated from both the auto- and cross-correlation functions while accounting for random changes in the interference pattern as it moves across pairs of receiving antennas.

[3] Theoretical and experimental studies of the performance of wind profilers are required to determine the limits of measurement accuracy and resolution of these remote sensing techniques to retrieve wind data. The performance of the Doppler method for wind measurement has been thoroughly studied and evaluated [*Doviak and Zrnic*, 1993]. The spaced antenna method, however, still requires a rigorous theoretical basis and experimental evaluation, even though the cross-correlation function

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has been derived based on wave scattering [*Doviak et al.*, 1996], and various wind estimators have been proposed [*Holloway et al.*, 1997]. A theoretical comparison, supported by simulations, of various estimators under the condition of large signal-to-noise-ratio (SNR) has been made by *Doviak et al.* [2004]. However, the performance of these wind estimators in the presence of noise has not been evaluated. The purpose of this paper is to extend the theory and simulations [*Zhang et al.*, 2003] to evaluate the performance of SA wind profilers under low SNR conditions. In the simulations, we follow the technique described by *May* [1988], and use a SNR = 0 dB for all our examples.

[4] The accuracy of an FCA estimator was first studied by *May* [1988], but his analysis is based upon an approximate formula for the standard deviation (*SD*) of the cross-correlation coefficient estimates. Other cross-beam wind estimators are (1) the cross- to auto-correlation-ratio (CACR) and (2) intersection (INT) methods [*Holloway et al.*, 1997], and (3) the slope-at-zero-lag (SZL) method [*Lataitis et al.*, 1995]. The theoretical accuracy of the baseline wind estimated with the latter two of these three estimators is derived by *Doviak et al.* [2004], who compare them with the theoretical accuracy obtained with the FCA estimator. The theoretical accuracies are also compared with those derived from simulations. A cross-correlation-ratio (CCR) method was recently proposed to measure cross-beam wind without the need for the auto-correlation function, and *Zhang et al.* [2003] compared its performance to that obtained with the FCA estimator. Except for *May*'s [1988] work, all the other theoretical error analyses for SA cross-beam wind estimates were performed under the assumption of infinite SNR. In practice, however, noise is usually an important component of the correlation functions and power spectra [*May et al.*, 1989; *Holdsworth*, 1999]. In this paper we focus our attention on the effects that additive white noise has on the FCA and CCR estimators examined by *Zhang et al.* [2003]. Although the focus of this paper is on the accuracy of the baseline wind component, the results can be applied to assess the errors in wind speed and direction, using the procedure of *Doviak et al.* [1976], to asses those errors in estimating the correlation lengths of refractive index perturbations (G. Hassenpflug et al., Standard deviation of refractive index perturbation correlation length estimates in spaced antenna observations, submitted to *Radio Science*, 2003), and to improve radar sensitivity of weak weather echoes in the presence of noise. Since FCA and CCR are unbiased estimators for small fluctuation of correlation estimates, we concentrate on standard deviations of wind estimates in this paper.

[5] The paper is organized as follows: In section 2, we provide a general background of the SA technique for wind estimation. In section 3, we present theoretical results for the *SD* of wind estimates in the presence of noise, and make comparisons with results from numerical simulations. In section 4, these results are then compared with root mean squared (rms) deviations obtain from data collected with NCAR's Multiple Antenna Profiling Radar (MAPR [*Brown et al.*, 1999; *Cohn et al.*, 1997, 2001]). Finally, we summarize results of SA wind estimation in the presence of noise, and discuss ways to reduce wind estimation errors.

2. Background

2.1. Auto- and Cross-Correlation Functions

[6] *Doviak et al.* [1996] derived the cross-correlation function of signals from spaced antennas for horizontally isotropic media and a receiver pair separation smaller than the horizontal dimension of the transmitting beam at the altitude of measurement. For a pair of receiving antennas separated by Δx, along the x-axis, the cross-correlation of signals separated by time-lag τ simplifies to

$$C_{12}^{(S)}(\tau) = S\exp\Big(-\beta_h^2(\Delta x/2 - v_{0x}\tau)^2 - \beta_h^2 v_{0y}^2\tau^2 - 2(k\sigma_t\tau)^2 - j\omega_d\tau\Big), \quad (1)$$

where the signal power is S, k is radar wave number, v_{0x} and v_{0y} are mean cross-beam wind components, σ_t is the radial component of turbulence on scales small compared to the resolution volume and the time required to make a wind estimate, ω_d is the mean Doppler shift, and β_h is inversely proportional to the diffraction (interference) pattern scale; $\beta_h \approx a_h$ for isotropic scattering from refractive index perturbations having a horizontal correlation length small compared to the transmitting antenna diameter, where a_h is a known constant calculated from antenna measurements [*Doviak et al.*, 1996]. These latter conditions are typically satisfied for profilers operating in the upper UHF bands.

[7] In general the complex voltages used to estimate the correlation functions contain signal and noise components, and hence the estimated auto- and cross-correlation functions are contaminated by noise power. Assuming that noise voltage is uncorrelated between contiguous voltage samples, and is uncorrelated between the two receivers (e.g., external noise is negligible compared to internally generated receiver noise), noise power N contributes only to the expected value (i.e., ensemble mean) of the auto-correlation at zero lag, and not to the expected value of the mean cross-correlation. Thus the expected values for auto- and cross-correlation functions in the presence of noise can be written in the Gaussian form:

$$C_{11}(\tau) = C_{11}^{(S)}(\tau) + N\delta(\tau) = S\exp\left(-\frac{\tau^2}{2\tau_c^2} - j\omega_d\tau\right) + N\delta(\tau), \quad (2)$$

$$C_{12}(\tau) = C_{12}^{(S)}(\tau) = S\exp\left(-\frac{\left(\tau-\tau_p\right)^2}{2\tau_c^2} - \eta - j\omega_d\tau\right) \equiv S\rho_{12}(\tau)\exp(-j\omega_d\tau), \quad (3)$$

where

$$\tau_c = \frac{1}{\beta_h\sqrt{2\left(v_{0x}^2+v_s^2\right)}},\quad \tau_p = \frac{v_{0x}\Delta x}{2\left(v_{0x}^2+v_s^2\right)},\quad \eta = \left(\frac{\beta_h\Delta x}{2}\right)^2\left(\frac{v_s^2}{v_{0x}^2+v_s^2}\right), \quad (4)$$

N is the noise power, $\rho_{12}(\tau)$ is the signal correlation coefficient, and $\delta(\tau) = 1$ if $\tau = 0$, otherwise $\delta(\tau) = 0$. The non-superscripted 'C's are the correlation functions for signal plus noise, whereas the signal-only correlation functions are superscripted with's'. The Gaussian parameters τ_c, τ_p and η specify signal coherence time, the time-lag to the cross-correlation peak, and the magnitude of cross-correlation peak relative to the signal auto-correlation at zero lag. It is noted that (3) is simply another form of (1) represented by Gaussian parameters. The term v_s, a de-correlating wind component that determines the correlation of the signals from the spaced receiving antennas,

$$v_s^2 = v_{0y}^2 + 2\left(\frac{k\sigma_t}{\beta_h}\right)^2 \quad (5)$$

depends on both the cross-baseline wind component v_{0y}, turbulence σ_t, as well as on the horizontal scale (correlation length) of the refractive index perturbations and antenna size (i.e., through the parameter β_h [*Doviak et al.*, 1996]).

[8] It is noted that N in (2) represents the white noise power in the data used to estimate the correlation functions. In some profilers, coherent averaging is used for which M_c samples of echoes from a fixed range are uniformly weighted and summed [*Gossard and Strauch*, 1983, section 11.2]. Whereas echo samples are spaced at intervals T_s equal to the transmitted Pulse Repetition Time (PRT), the separation of coherently averaged samples used to estimate the correlation functions is an effective sampling time, $T_{se} = T_s \times M_c$. This averaging is called "coherent" because the echo samples within the sum are typically correlated (i.e., $T_{se} \ll \tau_c$), whereas the noise samples are uncorrelated. In this case, N is the radar system noise power P_n divided by the number of coherently averaged samples M_c,

$$N = \frac{P_n}{M_c}, \quad (6)$$

with the radar system noise power given by [*Doviak and Zrnic*, 1993]

$$P_n = kT_{sy}B_n, \quad (7)$$

where k is the Boltzmann constant, T_{sy} the system temperature in degrees Kelvin, and B_n the noise bandwidth.

2.2. Cross-Correlation Ratio (CCR) Method

[9] Assume antennas 1, 2 are aligned along the baseline Δx (i.e., $\Delta y = 0$), and take the ratio of the magnitude of the cross-correlation function (1) at positive and negative lags and the natural logarithm of the ratio to obtain [*Zhang et al.*, 2003]

$$L(\tau) = \ln\frac{|C_{12}(\tau)|}{|C_{12}(-\tau)|} = \beta_h^2 2\Delta x v_{0x}\tau. \quad (8)$$

Therefore the natural logarithm of the CCR is linearly proportional to the mean transverse velocity v_{0x} along the baseline Δx. This result has a form similar to that of the slope-at-zero-lag method [*Lataitis et al.*, 1995]. The slope of $L(\tau)$ versus τ is proportional to the receiving antenna separation Δx, the baseline velocity v_{0x}, and β_h^2; the influences of turbulence and cross-baseline wind are canceled in the CCR method. Because of the linear relation, multilag data can be least squares fitted to a straight line to reduce the detrimental effects of receiver noise. The baseline wind component can be obtained by rewriting (8) as

$$v_{0x} = \frac{L(\tau)}{2a_h^2\Delta x\tau}. \quad (9)$$

[10] In (9) we have replaced the unknown parameter β_h with a_h, a parameter having a value that can be calculated from SA antenna measurements. This replacement is permissible if the Bragg scatterers' horizontal correlation lengths are small compared to the diameter of the transmitting antenna. This condition is usually satisfied for short wavelength profilers such as the MAPR (λ = 0.328 m); for MAPR, a_h = 1.6 m^{-1} [*Cohn et al.*, 1997].

2.3. Briggs' Full Correlation Analysis

[11] *Briggs et al.*'s [1950] and *Briggs*' [1984] FCA and its modified forms are the most commonly implemented methods for estimating cross-beam wind from SA measurements. The impetus for developing the FCA method was to use the auto-correlation function to account for the changes in the interference

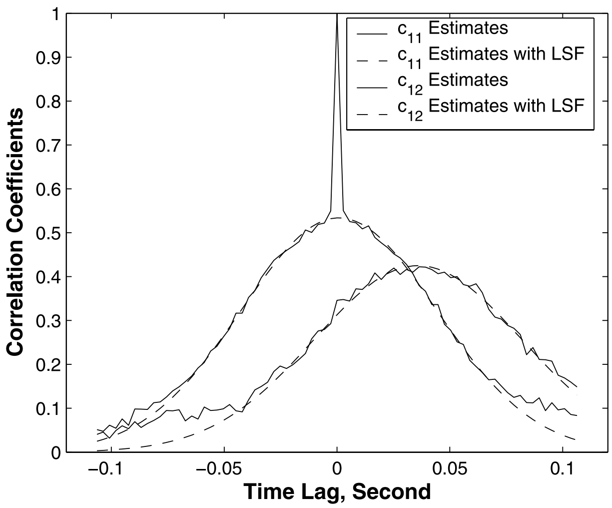

Figure 1. An example of estimated auto/cross-correlation coefficients of signals measured by MAPR at range of 1.6 km. Gaussian fitted parameters are $\hat{\eta} = 0.228$, $\hat{\tau}_c = 0.0433$ s, $\hat{\tau}_p = 0.0359$ s, $SNR = 0.588$ dB, $T_{se} = 0.0028$ s, $M = 900$, and $M_I = 32.9$.

pattern that caused bias in wind estimates based only on cross-correlation data, such as the spaced antenna drift (SAD) method. The FCA is generally a rather complicated procedure, but for horizontally isotropic scatterers it yields the following simple expression for the wind component parallel to the receiver pair baseline,

$$v_{0x} = \frac{\Delta x \tau}{2\tau_x^2} \tag{10}$$

where τ_x is the lag at which $|C_{11}(\tau)| = |C_{12}(0)|$. Contrary to the CCR method, no assumption about the horizontal correlation length of the refractive index perturbations is required when applying the FCA method.

[12] The derivation leading to (10) also does not require any assumption about the shape of the correlation functions. If τ_p and τ_x are estimated directly without assumption for the form of the correlation functions (as opposed to obtaining these lags from a least squares fitted Gaussian function), the estimator is called a Direct-FCA (i.e., D-FCA), and the equation to calculate the baseline wind variance is given by *Doviak et al.* [2004]. If the correlation functions are well represented by a Gaussian form, this equation reduces to a simple form. In this paper τ_p and τ_x are estimated using a D-FCA algorithm, and it is assumed that the correlation functions have a Gaussian form as suggested by MAPR data (Figure 1).

[13] A two-step procedure is used to estimate the correlation functions: (1) five blocks of M_t coherently averaged samples are used to estimate a set of correlation functions (but typically only the correlation function at M_{in} lags of interest are calculated, using data samples starting with the sample $M_{in} + 1$ and ending with the sample $M_t - M_{in}$), and (2) for real data, correlation function estimates from several blocks are averaged (block averaging is not used for simulated data). This procedure provides the same number, $M = 5 \times (M_t - 2 \times M_{in})$, of samples for the correlation function estimates (at each lag of interest) to be compared with theory, which also assumes an identical number of samples at each lag. To reduce the effects of imbalances between the receivers, the auto-correlation coefficient estimates (Figure 1) are the geometric average of the auto-correlation of signals measured by the two receivers.

[14] A Gaussian function (dashed line) has been least squares fitted (LSF) to correlation data at ±18 lags centered at the peaks in Figure 1 to show that the correlation functions are well represented by Gaussian forms. At lag zero, noise produces a sharp peak in the auto-correlation function having amplitude about equal to the signal power (i.e., SNR $\approx$ 0.59 dB). There are also high-frequency fluctuations in the correlation estimates due to noise that adds errors. A minor peak at zero-lag appears in the cross-correlation coefficient due to correlated noise in the two receivers. The fact that this peak is much smaller than the peak in the auto-correlation function confirms our contention that the internal noise of the receivers dominates external noise.

3. Performance of Wind Estimators

[15] In the work of *May* [1988], the theoretical development of an error analysis for the FCA wind estimates was based on signal statistics without noise; only in the numerical simulations was noise added. May hypothesized that the noise effect on wind estimation is due to a reduced correlation coefficient. He then simply applied the formulas derived for the signal-only case to the signal plus noise case by simply reducing the correlation coefficient. For example, the standard deviation of cross-correlation magnitude estimates for signal only is represented by [*Zhang et al.*, 2003]

$$SD\left[\left|\hat{C}_{12}^{(s)}(\tau)\right|\right] = \frac{S}{\sqrt{2M_I}}\left(1 + \rho_{12}^2(\tau)\right)^{1/2}. \tag{11a}$$

[16] On the basis of *May*'s [1988] hypothesis, the standard deviation of cross-correlation estimates with noise present would have the same form as (11a) but with a reduced correlation coefficient $\rho_{12}(\tau) \rightarrow$

$\rho_{12}(\tau)\frac{S}{S+N}$. Thus $SD[|\hat{C}_{12}(\tau)|]$ in the presence of noise is, according to May,

$$SD\left[\left|\hat{C}_{12}(\tau)\right|\right] = \frac{S}{\sqrt{2M_I}}\left(1+\left(\rho_{12}(\tau)\frac{S}{S+N}\right)^2\right)^{1/2}. \tag{11b}$$

This differs significantly from the rigorously derived formula presented in the next section.

3.1. Standard Error of Correlation Function Magnitude Estimates

[17] The variances of the auto- and cross-correlation magnitude estimates, derived in Appendix A, are

$$\begin{aligned}\mathrm{var}\left[\left|\hat{C}_{11}(\tau)\right|\right] = {} & \frac{1}{2M_I}\left(S^2+\left|C_{11}^{(S)}(\tau)\right|^2\right) \\ & + \frac{N}{M}\left(S+\left|C_{11}^{(S)}(2\tau)\right|\right)+\frac{N^2}{2M}(1+\delta(\tau))\end{aligned} \tag{12}$$

$$\mathrm{var}\left[\left|\hat{C}_{12}(\tau)\right|\right] = \frac{1}{2M_I}\left(S^2+\left|C_{12}^{(S)}(\tau)\right|^2\right)+\frac{NS}{M}+\frac{N^2}{2M} \tag{13}$$

where the number of independent signal samples within the dwell time $T_d = M \times T_{se}$ is $M_I = T_d/(\sqrt{\pi}\tau_c)$, and M is the total number of coherently averaged samples. As expected, the first terms are the noise-free signal contributions to the variances, and these are inversely proportional to M_I. The second and third terms are inversely proportional to M because noise samples at different lags are independent. If N and τ are zero, (12) gives the variance for estimated signal power (i.e., $\mathrm{var}[\hat{S}] = S^2/M_I$), and if $S = 0$, the noise power estimate variance decreases with the inverse of M as expected. Furthermore, (13) reduces to that result for the noise-free signal case (11a). If coherent averaging is not used, (12) and (13) would still apply with N replaced by P_n, and M replaced by M_cM, the total number of samples. Because $\frac{N}{M} = \frac{P_n}{MM_c}$, coherent averaging does not change the terms in (12) and (13) to first order in N. But if coherent averaging is used, the terms to second order in N are reduced by M_c. On the other hand, if coherent averaging is not used, there are about M_c times as many lags of correlation estimates that allow least squares fitting (LSF) over more data points; this would reduce the correlation function estimate variance associated with noise. Thus the principal, and likely the only, advantage for coherent averaging is the reduced size of time series data and the number of data points used in LSF.

[18] The covariance of the cross-correlation estimates at lags τ_1 and τ_2, and that between auto- and cross-correlation estimates derived in Appendix A are

$$\begin{aligned}\mathrm{cov}\left[\left|\hat{C}_{12}(\tau_1)\right|,\left|\hat{C}_{12}(\tau_2)\right|\right] = {} & \frac{1}{2M_I}\left[S^2\exp\left(-\frac{(\tau_2-\tau_1)^2}{4\tau_c^2}\right)\right. \\ & +\left|C_{12}^{(S)}(\tau_1)\right|\left|C_{12}^{(S)}(\tau_2)\right| \\ & \left.\cdot\exp\left(\frac{(\tau_2-\tau_1)^2}{4\tau_c^2}\right)\right] \\ & +\frac{N}{M}\left|C_{11}^{(S)}(\tau_1-\tau_2)\right| \\ & +\frac{N^2}{2M}\delta(\tau_1-\tau_2)\end{aligned} \tag{14}$$

$$\begin{aligned}\mathrm{cov}\left[\left|\hat{C}_{11}(\tau_1)\right|,\left|\hat{C}_{12}(\tau_2)\right|\right] = {} & \frac{1}{2M_I}\left|C_{11}^{(S)}(\tau_1)\right|\left|C_{12}^{(S)}(\tau_2)\right| \\ & \cdot\left[\exp\left(\frac{(\tau_2-\tau_1-\tau_p)^2}{4\tau_c^2}\right)\right. \\ & \left.+\exp\left(\frac{(\tau_2+\tau_1-\tau_p)^2}{4\tau_c^2}\right)\right] \\ & +\frac{N}{2M}\left[\left|C_{12}^{(S)}(\tau_2-\tau_1)\right|\right. \\ & \left.+\left|C_{12}^{(S)}(\tau_2+\tau_1)\right|\right]\end{aligned} \tag{15}$$

The first terms, the covariances of signal correlation magnitude estimates, are the same as that of (A14b) and (A15) in *Zhang et al.* [2003]. They are inversely proportional to the number of independent samples M_I. The other terms are contributions from noise, and they are inversely proportional to the number of samples M. The noise terms also depend on signal correlation magnitudes. The second-order term of noise power does not appear in the covariance of the auto- and cross-correlation magnitude estimates because noises at different receivers and time-lags are not correlated.

3.2. Standard Error of CCR Wind Estimates

[19] The cross-correlation ratio $L(\tau)$ at any time-lag $\tau \neq 0$ can be used to determine the baseline velocity v_{0x}. In practice, true $L(\tau)$ is unknown, and only estimates $\hat{C}_{12}(\tau)$ and $\hat{L}(\tau)$ are obtained from time series data. Following (9), we write the estimated transverse wind component $\hat{v}_{0x}$ as

$$\hat{v}_{0x} = \frac{\hat{L}(\tau)}{2a_h^2\Delta x\tau} = \frac{1}{2a_h^2\Delta x\tau}\ln\left(\frac{\left|\hat{C}_{12}(\tau)\right|}{\left|\hat{C}_{12}(-\tau)\right|}\right). \tag{16}$$

The variance of the estimation error for the transverse velocity is

$$\mathrm{var}[\hat{v}_{0x}] = \frac{1}{\left(2a_h^2 \Delta x \tau\right)^2} \mathrm{var}\left[\hat{L}(\tau)\right], \quad (17)$$

whereas the variance of the estimated cross-correlation ratio can be expressed as [*Papoulis*, 1965, section 7–3]

$$\mathrm{var}\left[\hat{L}(\tau)\right] = \frac{\mathrm{var}\left[\left|\hat{C}_{12}(\tau)\right|\right]}{\left|C_{12}(\tau)\right|^2} + \frac{\mathrm{var}\left[\left|\hat{C}_{12}(-\tau)\right|\right]}{\left|C_{12}(-\tau)\right|^2} - 2\frac{\mathrm{cov}\left[\left|C_{12}(\tau)\right|\left|\hat{C}_{12}(-\tau)\right|\right]}{\left|C_{12}(\tau)\right|\left|C_{12}(-\tau)\right|} \quad (18)$$

where the variance and covariance of the cross-correlation estimates are given by (13) and (14).

[20] Substituting (13) and (14) into (18), we have the variance of estimated $L(\tau)$ as follows:

$$\mathrm{var}\left[\hat{L}(\tau)\right] = \frac{1}{M_I} \cdot \left(1 - \exp\left(\frac{\tau^2}{\tau_c^2}\right) + \frac{1}{\rho_{12}^2(0)} \cdot \left(\exp\left(\frac{\tau^2}{\tau_c^2}\right)\cosh\left(\frac{2\tau\tau_p}{\tau_c^2}\right) - 1\right)\right) + \frac{1}{M}\left(\frac{2N}{S} + \frac{N^2}{S^2}\right)\frac{1}{\rho_{12}^2(0)}\exp\left(\frac{\tau^2}{\tau_c^2}\right) \cdot \cosh\left(\frac{2\tau\tau_p}{\tau_c^2}\right) - \frac{2}{M}\frac{N}{S}\frac{1}{\rho_{12}^2(0)}\exp\left(-\frac{\tau^2}{\tau_c^2}\right) \quad (19a)$$

If the time-lag is much smaller than the coherence time (i.e., $\tau \ll \tau_c$), and if τ_p is about equal to or less than τ_c (a condition that results if Δx is less than D), (19a) reduces to

$$\mathrm{var}\left[\hat{L}(\tau)\right] \approx \frac{\tau^2}{\tau_c^2 M_I} \cdot \frac{1}{\rho_{12}^2(0)}\left(1 + 2\frac{\tau_p^2}{\tau_c^2} - \rho_{12}^2(0)\right) + \frac{4}{M}\frac{N}{S}\frac{1}{\rho_{12}^2(0)}\frac{\tau^2}{\tau_c^2}\left(1 + \frac{\tau_p^2}{\tau_c^2}\right) + \frac{1}{M}\frac{N^2}{S^2}\frac{1}{\rho_{12}^2(0)} \quad (19b)$$

where $\rho_{12}(0)$ is the cross-correlation coefficient at zero-lag. Besides being inversely proportional to the number of independent samples M_I, the noise-free first term in (19b) decreases as the cross-correlation coefficient $\rho_{12}(0)$ increases because estimates of $\rho_{12}(\pm\tau)$ on either side of $\rho_{12}(0)$, and $\rho_{12}(0)$ itself, are highly correlated if $\tau \ll \tau_c$, and thus fluctuations in their estimates do not significantly affect $\hat{L}(\tau)$. The second term, however, is inversely proportional to the number of samples M and does not vanish as signal correlation $\rho_{12}(0) \rightarrow 1$. This means that for the finite SNR case, small antenna separation and lag spacing are not optimal as they are in infinite SNR case as shown by *Zhang et al.* [2003].

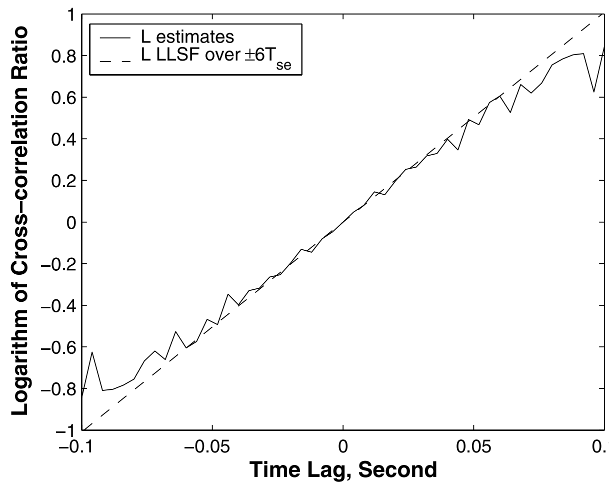

Figure 2. Estimated natural logarithm of the cross-correlation ratio for simulated signals. Parameters used in simulations are $\rho_{12}(\tau_p)$ = 0.7, τ_c = 0.0522 s, τ_p = 0.012 s, SNR = 0 dB, $T_{se} = T_s$ = 0.004 s, M = 8036, and M_I = 348.

[21] To verify the theory, we ran numerical simulations using the technique described by *May* [1988]. An example of the logarithm of cross-correlation ratio estimates ($\hat{L}(\tau)$) for simulated signals with noise present is shown in Figure 2. A SNR = 0 dB is used in the simulations. We can see the noise contribution as high frequency fluctuations about the fitted line; the least squares fitted line for the center portion of the points (i.e., 13 points) is shown as the dashed line. Hence the cross-correlation ratio can be estimated from data either at a pair of positive and negative lags small compared to τ_c, for example at $\pm T_{se}$ (we call this the two-lag CCR estimator, for which T_{se} is the spacing between coherently averaged samples), or from a local least squares fit (LLSF) line (this is called the LLSF-CCR estimator). In this context, ''local'' defines the number of lags used in the LSF to be small compared to the correlation time τ_c. The two-lag CCR estimator for cross-beam wind measurements is analogous to the pulse pair estimator for radial wind measurements [*Doviak and Zrnic*, 1993]. *Doviak et al.* [2004] show that, under the condition $T_{se} \ll \tau_c$, and in the absence of noise, the two-lag CCR estimator is identical to the slope-at-zero-lag (SZL) estimator described by *Lataitis et al.* [1995]. It can be expected that the estimates obtained from a fitted line have smaller errors than those obtained by simply using data at $\pm T_{se}$. While the theory developed in this paper is limited to the two-lag CCR estimator, a simulation result shown next will confirm this expectation.

[22] The standard deviation of $\hat{L}(\tau)$ (i.e., $SD[\hat{L}(\tau)]$), obtained by taking the square root of (19a), is plotted in Figure 3 and compared with results from the numerical simulations. The theoretical result for infinite SNR case is also shown for comparison as well as those results obtained if May's hypothesis is used. As shown in the figure, the theoretical results agree well with those obtained from simulations. As expected, the standard error of $\hat{L}(\tau)$ in the presence of noise is larger than that without noise. For comparison we have also plotted the $SD[\hat{L}(\tau)]$ for $\hat{L}(\tau)$ calculated by a local least squares fitting of data, over thirteen-lags centered at zero-lag, to a straight line. Herein, "local" means that the span of lags used in the LSF is small compared to the correlation time τ_c. It is interesting and of practical use that for signals strongly contaminated by white noise (i.e., SNR = 0 dB), estimates of L made using the LLSF-CCR method have a SD about equal to that for the 2-lag CCR estimator for the noise-free case. But, neither the $SD[\hat{L}(\tau)]$ for $\hat{L}(\tau)$ estimated from data at two lags, nor the $SD[\hat{L}(\tau)]$ obtained using the LLSF-CCR method agree with the $SD[\hat{L}(\tau)]$ if May's hypothesis is used to account for noise. This is because the sampling errors of correlation estimates for signal-only are correlated from lag to lag [*Zhang et al.*, 2003] while the noise errors are independent of each other.

[23] Substituting (19) into (17), we obtain standard derivation of the velocity estimates

$$SD\left[\hat{v}_{0x}^{(CCR)}\right] = \frac{1}{2a_h^2\Delta x\tau}\left[\frac{1}{M_I}\left(1-\exp\left(\frac{\tau^2}{\tau_c^2}\right)\right.\right. + \frac{1}{\rho_{12}^2(0)}\left.\left(\exp\left(\frac{\tau^2}{\tau_c^2}\right)\cosh\left(\frac{2\tau\tau_p}{\tau_c^2}\right)-1\right)\right) + \frac{1}{M}\left(\frac{2N}{S}+\frac{N^2}{S^2}\right)\frac{1}{\rho_{12}^2(0)}\exp\left(\frac{\tau^2}{\tau_c^2}\right) \cdot\cosh\left(\frac{2\tau\tau_p}{\tau_c^2}\right) - \frac{2N}{MS}\frac{1}{\rho_{12}^2(0)}\exp\left(-\frac{\tau^2}{\tau_c^2}\right)\right]^{\frac{1}{2}} \tag{20a}$$

and if $\tau \ll \tau_c$, we obtain to second order in τ,

$$SD\left[\hat{v}_{0x}^{(CCR)}\right] = \frac{1}{2a_h^2\Delta x\tau_c\rho_{12}(0)}\cdot\left[\frac{1}{M_I}\left(1+2\frac{\tau_p^2}{\tau_c^2}-\rho_{12}^2(0)\right) + \frac{4}{M}\frac{N}{S}\left(1+\frac{\tau_p^2}{\tau_c^2}\right)+\frac{\tau_c^2}{\tau^2 M}\frac{N^2}{S^2}\right]^{1/2}. \tag{20b}$$

[24] We calculate the standard error of estimating transverse velocity using the CCR method. Using (20a), the standard error is plotted in Figure 4 as a function of antenna separation and time-lag for various SNR = 0, 5.0, 10.0 dB and infinite SNR. The assigned meteorological and radar parameters are given in the figure caption. As expected, the estimation error increases as SNR decreases. But in contrast to the infinite SNR case, the error becomes infinitely large as antenna separation and the time-lag spacing approach zero. This is reasonable because we do not expect a fixed beam single antenna receiver be able to measure the baseline wind component, unless N = 0, and σ_t = v_{oy} = 0 (i.e., scatterers are uniformly advected by wind parallel to the baseline). This suggests that an optimum antenna separation depends on noise level and time-lag spacing.

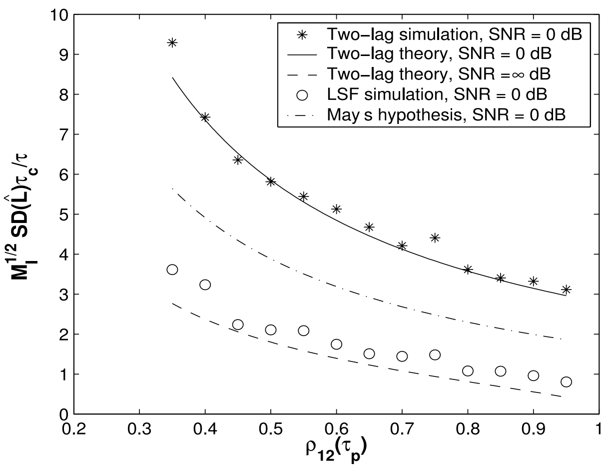

Figure 3. Standard derivation of the estimated cross-correlation ratio as a function of peak cross-correlation coefficient: comparison between simulation and theory. Parameters are τ_c = 0.0522 s, τ_p = 0.012 s, T_{se} = 0.004 s, and 200 realizations.

3.3. Standard Error of D-FCA Wind Estimates

[25] Because parameters τ_p and τ_x in (10) can only be estimated, the baseline wind estimate is expressed as

$$\hat{v}_{0x} = \frac{\Delta x\hat{\tau}_p}{2\hat{\tau}_x^2}. \tag{21}$$

Doviak et al. [2004] show that the correlation between the estimates $\hat{\tau}_p$ and $\hat{\tau}_x$ can be ignored in computing the standard deviation of the D-FCA baseline wind estimates. Thus the $SD[\hat{v}_{ox}]$ for baseline wind estimates obtained from the D-FCA is,

$$SD\left[\hat{v}_{0x}^{(D-FCA)}\right] = v_{0x}\left(\frac{\mathrm{var}[\hat{\tau}_p]}{\tau_p^2}+4\frac{\mathrm{var}[\hat{\tau}_x]}{\tau_x^2}\right)^{1/2}. \tag{22}$$

[26] Under the assumption that the correlation functions have a Gaussian form, the expressions for the variances of $\hat{\tau}_p$ and $\hat{\tau}_x$, derived by *Zhang et al.* [2003], are

$$\mathrm{var}\left[\hat{\tau}_p\right] = \left(\frac{\tau_c^2}{T_s}\right)^2 \left(\frac{\mathrm{var}\left[\left|\hat{C}_{12}(\tau_1)\right|\right]}{\left|C_{12}(\tau_1)\right|^2} + \frac{\mathrm{var}\left[\left|\hat{C}_{12}(\tau_2)\right|\right]}{\left|C_{12}(\tau_2)\right|^2} - 2\frac{\mathrm{cov}\left[\left|\hat{C}_{12}(\tau_1)\right|\left|\hat{C}_{12}(\tau_2)\right|\right]}{\left|C_{12}(\tau_1)\right|\left|C_{12}(\tau_2)\right|}\right) \quad (23)$$

$$\mathrm{var}[\hat{\tau}_x] = \frac{\tau_c^4}{\tau_x^2\left|C_{12}^2(0)\right|}\left(\mathrm{var}\left[\left|\hat{C}_{11}(\tau_x)\right|\right] + \mathrm{var}\left[\left|\hat{C}_{12}(0)\right|\right] - 2\mathrm{cov}\left[\left|\hat{C}_{11}(\tau_x)\right|, \left|\hat{C}_{12}(0)\right|\right]\right). \quad (24)$$

Substituting (13) and (14) into (23), we obtain the variance of $\hat{\tau}_p$ given by

$$\mathrm{var}\left[\hat{\tau}_p\right] = \frac{\tau_c^2}{4M_I}\frac{1-\rho_{12}^2(\tau_p)}{\rho_{12}^2(\tau_p)} + \frac{\tau_c^4}{\tau^2 M\rho_{12}^2(\tau_p)}\frac{N^2}{S^2} + \frac{\tau_c^2}{M\rho_{12}^2(\tau_p)}\frac{N}{S}. \quad (25)$$

[27] Using (12), (13), and (15) in (24), we have

$$\mathrm{var}[\hat{\tau}_x] = \frac{\tau_c^4}{\tau_x^2\rho_{12}^2(0)}\left[\frac{1}{M_I}\left(1+\rho_{12}^2(0) - 2\rho_{12}^2(0). \cdot \exp\left(\frac{\tau_x^2+\tau_p^2}{4\tau_c^2}\right)\cosh\left(\frac{\tau_x\tau_p}{2\tau_c^2}\right)\right) + \frac{N}{MS}(2+\rho_{11}(2\tau_x) - \rho_{12}(\tau_x) - \rho_{12}(-\tau_x)) + \frac{N^2}{MS^2}\right]. \quad (26)$$

Hence, when noise is present, (22), (25), and (26) constitute the theoretical analysis for errors in velocity retrieval using the D-FCA algorithm.

[28] Although a Gaussian form for the correlation functions is assumed in deriving (25) and (26), a Direct-FCA estimator does not require, in general, an assumed functional form for the correlation functions, but at least three lags of data about the lag being estimated are required [*Doviak et al.*, 2004]. In absence of noise, *Doviak et al.* [2004] show estimates that are obtained from a three-lag estimator have identical variance to those obtained with the two-lag estimator if the form of the correlation functions is Gaussian. However, as will be shown, in the presence of noise, increasing the number of lags about the lag being estimated using a LLSF to a polynomial improves significantly the estimate accuracy. Herein, the Direct-FCA estimators are defined as local fits to data about the lags being estimated (i.e., τ_p, τ_x). In contrast, the two-lag D-FCA, for which the theory applies, uses data at two contiguous lags about the cross-correlation peak to estimate τ_p and, to estimate τ_x, two lags about which the auto-correlation function equals the cross-correlation at zero lag [*Zhang et al.*, 2003]. To compare the theoretical performance for the two-lag estimators with the performance of LLSF estimators obtained from simulations, we fit a second-order

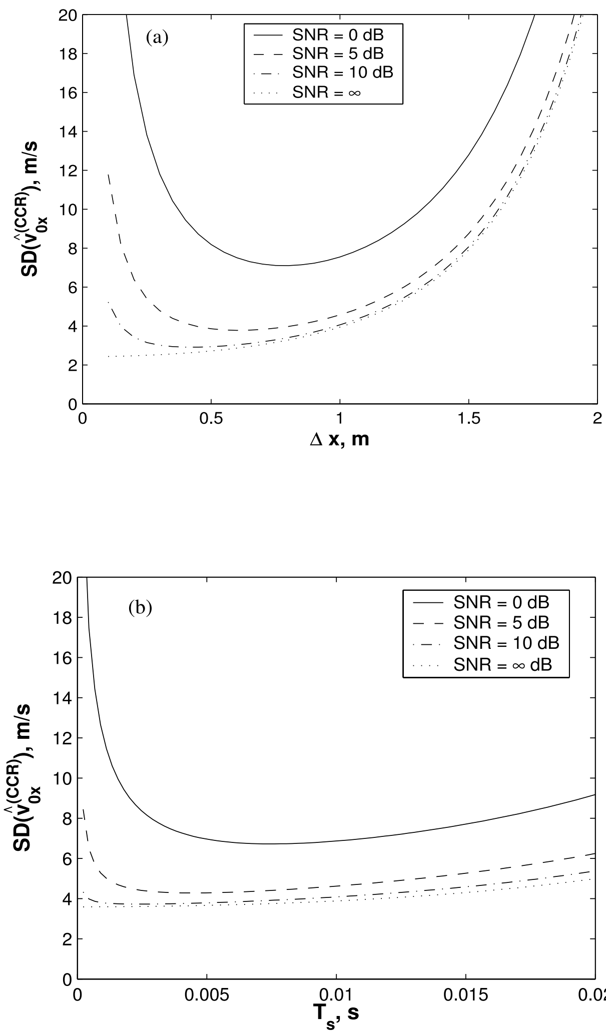

Figure 4. Standard deviations of baseline wind computed using the two-lag CCR estimator as a function of (a) receiving antenna separation for lag spacing of $T_{se} = T_s = 4$ ms, (b) time-lag spacing for $\Delta x = 0.5D = 0.92$ m. Other parameters are $v_{0x} = 10.0$ m/s, $\sigma_t = 1$ m/s, $v_{0y} = v_{0z} = 0$ m/s, $a_h = 1.6$ m^{-1}, $\lambda = 0.328$ m, $\tau_c = 0.022$ s, and dwell time $T_d = MT_{se} = 1$ s.

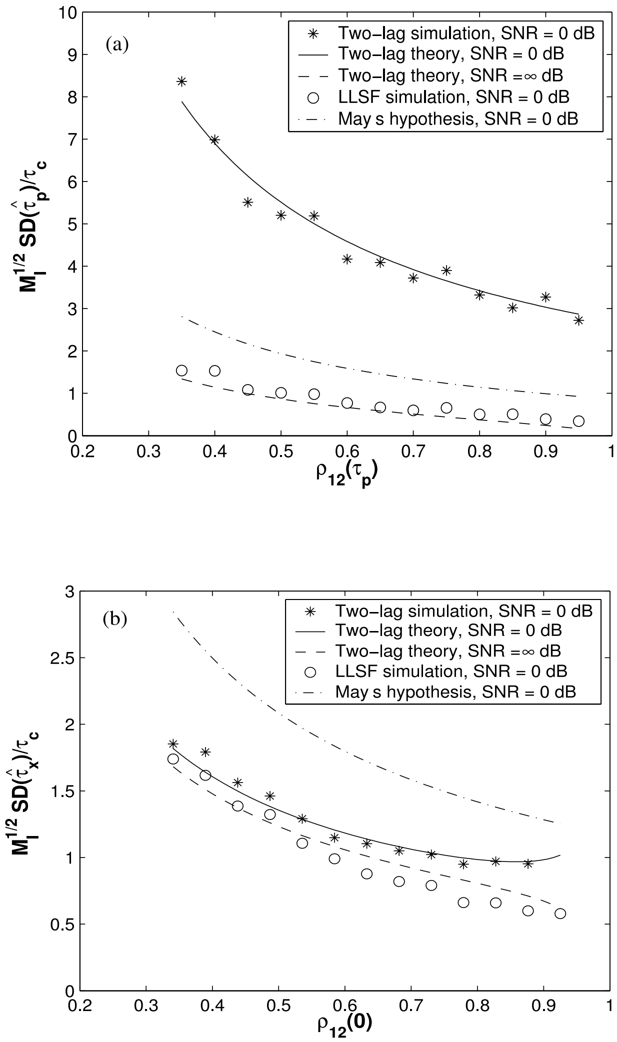

Figure 5. Comparison of theory (solid line) and simulation (asterisks) for the normalized SD of $\hat{\tau}_p$ and $\hat{\tau}_x$ obtained from two-lag D-FCA estimators: (a) SD[$\hat{\tau}_p$], (b) SD[$\hat{\tau}_x$]; $\tau_c = 0.0522$ s, $\tau_p = 0.012$ s, $T_{se} = 0.004$ s, and 200 realizations. Also plotted are the SD for the LLSF D-FCA estimates, results using May's hypothesis, and the theoretical SD for the two-way D-FCA estimates if SNR = ∞.

polynomial to the logarithm of correlation data at thirteen lags centered about τ_p and τ_x under the assumption that $13{\cdot}T_{se} < \tau_c$; this approach is called a LLSF D-FCA estimator.

[29] Figure 5 shows the variance of $\hat{\tau}_x$ and $\hat{\tau}_p$ calculated from (25) and (26) for SNR = 0 dB and, as well, the theoretical results if SNR = ∞. Also plotted in this figure are results for the simulated two-lag estimator (SNR = 0 dB) showing good agreement with theory. We have added to Figure 5 the standard deviations of the estimates obtained with the LLSF D-FCA estimator. The simulation results using the LLSF D-FCA estimator for SNR = 0 dB show standard deviations of the estimates to be reduced substantially as expected. Surprisingly, the performance of the LLSF D-FCA for SNR = 0 is as good as the performance of the two-lag D-FCA method for SNR = ∞. Using the LLSF D-FCA estimator, there is a substantial reduction in the variance of $\hat{\tau}_p$ compared to the much smaller reduction in the variance of $\hat{\tau}_x$. This is because the lag-to-lag change of cross-correlation of signals around the peak is smaller than that of the noise fluctuation while the change of the signal auto-correlation around τ_x is dominant over the noise fluctuation. It is noted that the standard errors of $\hat{\tau}_p$ and $\hat{\tau}_x$ estimated with the two-lag D-FCA or the 13 lag LLSF D-FCA methods do not agree with results obtained using *May*'s [1988] hypothesis.

[30] The standard deviation of baseline wind, for the two-lag D-FCA estimator, is calculated using (22); results are plotted in Figure 6. These results should be compared with the standard deviation of baseline wind estimates obtained using the two-lag CCR method (Figure 4); the radar and wind parameters used for the calculation are the same as that in Figure 4. For all SNR, the standard deviation of the D-FCA baseline wind estimates is larger than that obtained using the CCR method if Δx is less than $0.5D$; this is due to a large error in estimating τ_p. Figures 4 and 6 are also verified by *Doviak et al.* [2004] and *Zhang et al.* [2003] for the infinite SNR case, the error of wind measurements obtained using the two-lag D-FCA estimator is smaller than that obtained with the two-lag CCR estimator if receiving antenna separation Δx is larger than about $0.5D$, but the reverse is true if Δx is less than $0.5D$.

4. Verification With MAPR Data

[31] Data collected on October 4, 2003 with the MAPR at Boulder, Colorado, are also used to verify the theory developed in this paper. MAPR was vertically up-looking and the data used for the correlation analysis are for a range at 1.6 km. MAPR has four closely spaced antennas that are combined to form one transmitting antenna, and then act separately as four receiving antennas so that spaced antenna techniques can be applied. The MAPR transmitted 1.4 μs wide pulses with a PRT = T_s = 25 μs, and M_c = 112 uniformly weighted signal samples were averaged to give an effective lag spacing T_{se} = 2.8 ms between the coherently averaged samples. The time series of the coherently averaged samples is then used to estimate the correlation functions (e.g., Figure 1). As noted in section 2.1, N refers to the noise power that remains after coherently averaging 112 samples. There-

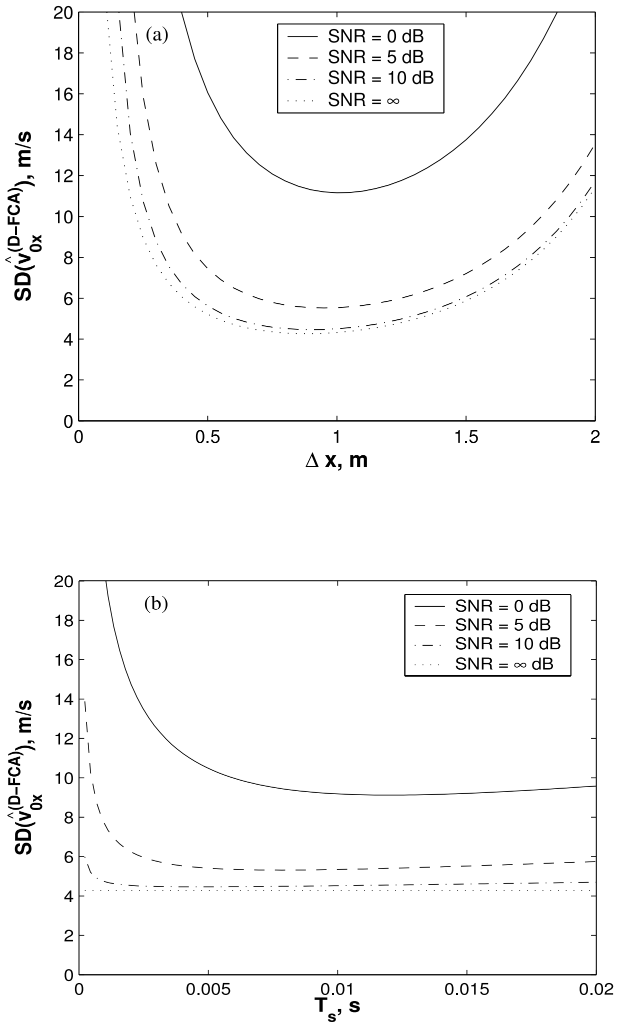

Figure 6. Standard error of the two-lag D-FCA baseline wind estimates as a function of (a) receiving antenna separation for lag spacing T_s = 4 ms, (b) time lag spacing for antenna separation of $\Delta x = 0.5D = 0.92$ m. Other parameters are $v_{0x} = 10.0$ m/s, $\sigma_t = 1$ m/s, $v_{0y} = v_{0z} = 0$ m/s, $a_h = 1.6$ m^{-1}, $\lambda = 0.328$ m, $\tau_c = 0.022$ s, and dwell time $T_d = 1$ s.

fore, the signal-to-noise ratios before and after coherent averaging are related as SNR(before) = SNR(after) − 10log(M_c).

[32] The two-step procedure described in Section 2.3 is used in the analysis of MAPR data, but correlation estimates from five blocks of M_t = 256 samples are averaged. Although $M = 5 \times (M_t - 2 \times M_{in}) = 900$ samples are used to estimate the correlation at each lag of interest, the number M_I of independent samples, in the effective dwell time $T_d = M \times T_{se} = 2.52$ s, is about $M_I = 26.5$ for the mean correlation time of about 54 ms (Figure 7).

[33] We also have calculated time-lags (τ_p, τ_x), the logarithm of the cross-correlation ratio, and wind from estimates of the Gaussian parameters (η, τ_c, and τ_p); this wind estimator is labeled G-FCA [*Doviak et al.*, 2004]. The Gaussian parameter estimates are obtained by least squares fitting the logarithm of auto- and cross-correlation data at ±18 lags about the peaks of C_{11} and C_{12}, excluding the data point at zero-lag. The signal power is obtained from the fitted auto-correlation. The noise power is the difference between the auto-correlation data at lag zero, minus the signal power. The results for 50 realizations starting at 17:03:50 are shown in Figure 7. For these 50 estimates, the mean signal-to-noise ratio is about SNR = −0.86 dB, and the means of the estimated Gaussian parameters are: $\eta = 0.267$, $\tau_c = 0.0537$ s, and

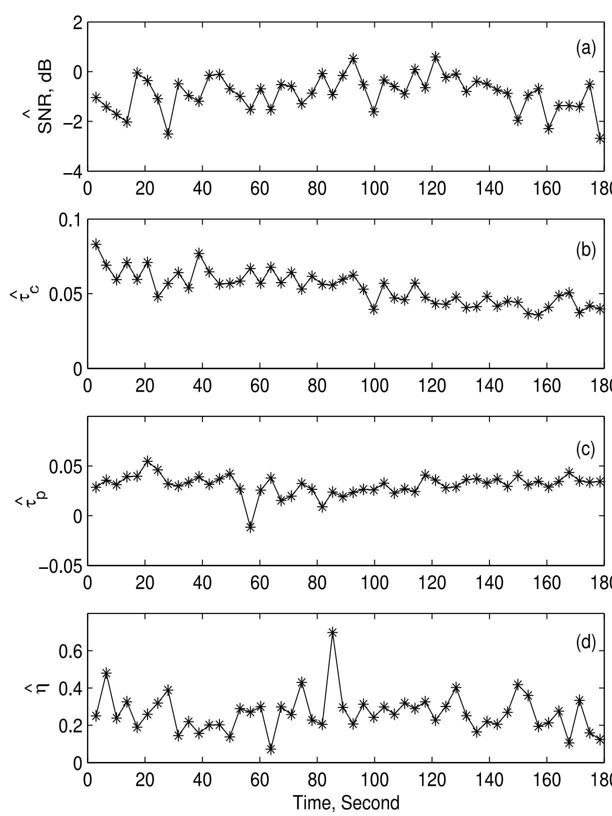

Figure 7. Time series plots of estimated Gaussian parameters from the signals measured by MAPR at range of 1.65 km and using the G-FCA algorithm. The starting time is 17:03:50 on 4 October 2003. The mean of the estimated parameters are SNR = −0.86 dB, $\eta = 0.267$, $\tau_c = 0.0537$ s, and $\tau_p = 0.0310$ s. Other parameters are $T_{se} = 0.0028$ s, $M = 900$, and $M_I = 26.5$.

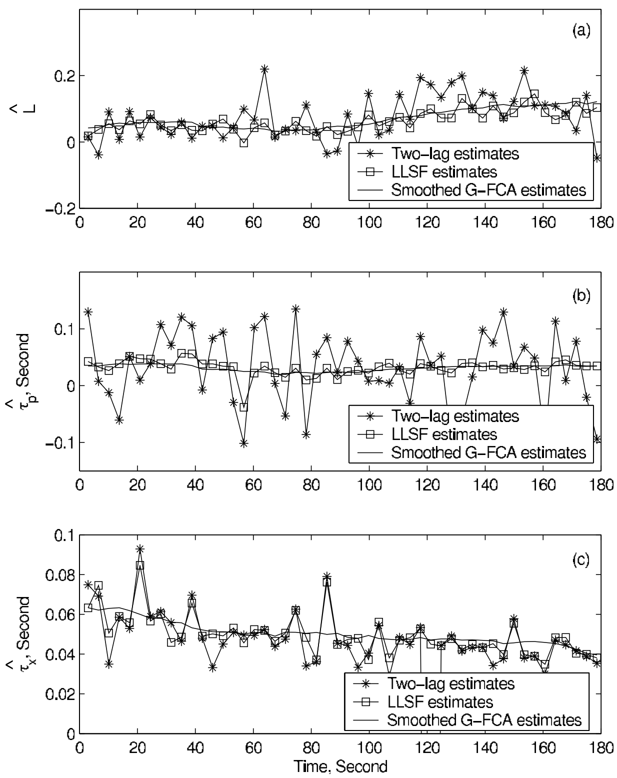

Figure 8. Time series plots of the estimated parameters of CCR and D-FCA estimators for the signals measured by MAPR at range of 1.6 km. The starting time is 17:03:50 on 4 October 2003.

τ_p = 0.0310 s. These mean values for the SNR and mean Gaussian parameters are used in the expressions to calculate the theoretical standard deviation of the estimates $\hat{L}$, $\hat{\tau}_p$, $\hat{\tau}_x$, and $\hat{v}_{ox}$ in the two-lag CCR and D-FCA methods.

[34] The parameters $\hat{L}$, $\hat{\tau}_p$, $\hat{\tau}_x$ used for wind estimation are obtained using the two-lag estimators as well as a LLSF method. For the two-lag CCR estimator, the logarithm L of cross-correlation ratio is estimated using the cross-correlation data at lags $\pm T_{se}$, and the SD of the estimates are calculated to compare with the theory. In addition the LLSF-CCR estimator has been evaluated. The LLSF-CCR estimator fits a line to $\hat{L}$ at fifteen (from $-7T_{se}$ to $+7T_{se}$) contiguous lags about zero-lag. For the D-FCA estimator, $\hat{\tau}_p$ and $\hat{\tau}_x$ are directly estimated from auto- and cross-correlation data using two-lags, as well as a LLSF to fifteen contiguous lags (omitting zero-lag of auto-correlation) about the first guess. We use a LLSF D-FCA to emphasize that we do not need to know the form of the correlation functions in applying the FCA method, and to show how the use of data at more lags can improve estimate accuracy, especially if noise is significant.

[35] $\hat{L}$, $\hat{\tau}_p$, and $\hat{\tau}_x$, estimated with two-lag and LLSF methods, are shown in Figure 8. The G-FCA estimates smoothed with moving average over nine data points (28.7 s) are also shown for reference. The smoothed G-FCA estimates show a meteorological change over the 180 s data period (e.g., note the monotonic decrease of $\hat{\tau}_x$). We assume that changes on time scales larger than about 30 s are associated with meteorological variance, whereas changes on smaller scales are due to estimate variance. After removing the variance associated with the meteorological change by subtracting moving average values from the estimated parameters, the standard deviations of the parameters are calculated. The normalized standard deviations of estimates obtained with the two-lag CCR, the two-lag D-FCA, as well as the fifteen-lag LLSF-CCR and LLSF D-FCA methods to estimate wind, are listed in Table 1. Standard deviations calculated from a correlation analysis of the parameter estimates are also listed in Table 1; these are superscripted with asterisks. The asterisked standard deviations are obtained by taking the square root of the difference of zero-lag and plus/minus one lag of the auto-correlation functions of the estimated parameters. The standard deviations obtained from the correlation analysis agree well with that obtained with the moving average method. As might be expected, the *SD* of the two-lag estimates is larger than that obtained with the fifteen-lag least squares fitting methods. If we compare the two-lag estimators with the theory including noise (i.e., SNR = −0.86 dB), the *SD* agrees reasonably well. Comparing the right two columns of Table 1, we note that the local 15-lag LLSF estimators

Table 1. Standard Deviation of Parameter Estimates for MAPR Data

Normalized SD of Parameter Estimates	Two-Lag Estimators: Data (SNR = −0.86dB)	Two-Lag Estimators: Theory (SNR = −0.86dB)	Fifteen-Lag LLSF Data (SNR = −0.86dB)	Two-Lag Theory (No Noise)
$M_I^{1/2}SD[\hat{L}]\tau_c/T_{se}$	6.14/5.82*	6.47	1.96/1.97*	1.72
$M_I^{1/2}SD[\hat{\tau}_p]/\tau_c$	6.21/5.94*	5.25	0.76/0.72*	0.63
$M_I^{1/2}SD[\hat{\tau}_x]/\tau_c$	1.05/1.15*	0.95	0.67/0.77*	0.82

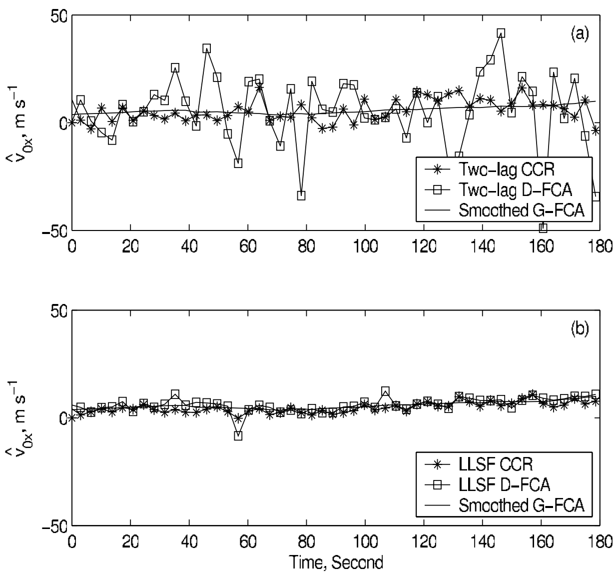

Figure 9. Time series plots of the baseline wind estimates using CCR and D-FCA estimators for the signals measured by MAPR at range of 1.6 km. The starting time is 17:03:50 on 4 October 2003.

produce estimate variance, for SNR = −0.86 dB, about equal to the estimate variance given by the theory for two-lag estimators if SNR = ∞.

[36] The baseline wind, estimated using the two-lag and the LLSF CCR and D-FCA methods, are shown in Figure 9 along with the wind calculated using smoothed G-FCA parameter estimates. The *SD* of baseline wind estimates is given in Table 2. The listed *SD*s are that calculated after the subtraction of the smoothed G-FCA estimates and those with asterisks by a correlation analysis to remove the meteorological variance. The theoretical *SD* calculated for the two-lag CCR estimator agrees reasonably well with the *SD* obtained from data, but the agreement for the two-lag D-FCA estimator is not as good. This poorer performance for the two-lag estimator is attributed to the large variance in estimating τ_p. As expected, at low SNR, the two-lag estimators give significantly larger deviations in wind estimates than the fifteen-lag LLSF estimators. For reference, the *SD* of wind estimates using the G-FCA method [*Doviak et al.*, 2004] containing the meteorological variance is $SD(\hat{v}_{0x}^{(G\text{-}FCA)})$ = 2.19 m/s and that after removing the meteorological change using moving average and correlation analysis are 1.56/1.64* m/s, respectively.

5. Summary and Conclusions

[37] We extended the theory of error analysis for wind estimates using the SA technique to include additive white noise (that for signal contaminated by clutter is the next to study). Using a rigorous derivation, the developed theory shows the effect of noise on SA wind estimates is different from that hypothesized by *May* [1988] using correlation coefficients reduced by noise. The theory is verified with simulations and MAPR data. Our results show that if noise is present there are optimal spacings, Δx and T_{se}, for the receiving antennas and coherently average samples when a two-lag direct estimator is used. The parameters estimated with least squares fitting have much smaller errors, as expected, compared to those obtained from the two-lag estimators. Although we restrict our theoretical study to the two-lag CCR and D-FCA wind estimators, our results suggest that estimating the Gaussian parameters η, τ_c, τ_p (either from (1) two lags about the correlation function peaks (i.e., the G-FCA estimator [*Doviak et al.*, 2004]) for the case of large SNR, or (2) a multilag LSF of a Gaussian function to the correlation coefficients weighted strongly about the correlation peaks for the case of low SNR) and using those estimates to calculate baseline wind, would result in the smallest errors. *Doviak et al.* [2004] show, for the case SNR = ∞, that (1) or (2) achieve about the same results. For antenna separation much less than D/2, the CCR estimator performs as well as G-FCA estimator for all SNR.

[38] The standard deviations of the Gaussian parameter and baseline wind estimates for MAPR data are slightly different from that for simulated data and theory. This is because atmospheric echoes are not exactly a stationary Gaussian random process, and they may contain clutter.

Table 2. Standard Deviation of Baseline Wind Estimates for MAPR Data

SD of Wind Estimates, m/s	Two-Lag Estimators: Data (SNR = −0.86 dB)	Two-Lag Estimators: Theory (SNR = −0.86 dB)	Fifteen-Lag LLSF Data (SNR = −0.86 dB)	Two-Lag Theory (No Noise)
$SD[\hat{v}_{0x}^{(CCR)}]$	4.81/4.40*	4.89	1.59/1.53*	1.30
$SD[\hat{v}_{0x}^{(D-FCA)}]$	16.64/16.16*	10.26	2.67/2.66*	2.09

Nevertheless, the theoretically calculated standard deviations of SA wind estimates for MAPR are in approximate agreement with theory (i.e., 4.81 versus 4.89 m/s for the two-lag CCR estimator and 16.64 versus 10.26 m/s for the two-lag D-FCA estimator). The relative large difference between the two-lag D-FCA estimator and the theory could be due to a few outliers and the large variance of τ_p estimates. The standard deviations of wind estimates using LLSF methods, applied to the case for which data have a SNR = −0.86 dB, are close to the theoretical results for the case without noise present. This means that the noise effects on wind estimates can be effectively reduced by multilag LLSF estimation. The performance of multilag LLSF and Gaussian estimators has little dependence on time-lag spacing T_{se} as long as there are enough data points within correlation time to capture the shape of correlation functions (i.e., $T_{se} \ll \tau_c$, the correlation time).

[39] By examining equations (12) and (13), it is seen that coherent averaging makes little difference in reducing the deviations of the correlation estimates: no difference in the first-order noise term and a factor of M_c in the second-order noise term. On the other hand, there would be a smaller lag-spacing (i.e., data at more lags) that allows more effective LSF methods to reduce the effects of noise in estimating wind. It is true that coherent averaging improves signal SNR. However, power estimation by multilag LSF (excluding zero-lag) to auto-correlation functions of signals without coherent averaging can be more accurate than that from auto-correlation function of coherently averaged signals at zero-lag subtracting mean noise. Thus the principal, and likely the only, advantage for coherent averaging is the reduced data size. This is reasonable because estimating correlation functions with more samples can also suppress noise. A reasonable choice for M_c is that the lag spacing be a tenth of correlation time (i.e., $T_{se} = M_c T_s \approx \tau_c/10$).

Appendix A: Variance and Covariance of the Magnitudes of Correlation Estimates in the Presence of Noise

[40] The magnitudes of correlation function estimates are used in the SA technique to estimate wind. Hence the variance and covariance of the magnitude of cross-correlation estimates are needed to assess the performance of the wind estimators. For example, the covariance of the cross-correlation function for signal plus noise at lags τ_1, τ_2 is

$$\mathrm{cov}\left[\left|\hat{C}_{12}(\tau_1)\right|, \left|\hat{C}_{12}(\tau_2)\right|\right] = \langle\left|\hat{C}_{12}(\tau_1)\right|\left|\hat{C}_{12}(\tau_2)\right|\rangle - \langle\left|\hat{C}_{12}(\tau_1)\right|\rangle\langle\left|\hat{C}_{12}(\tau_2)\right|\rangle, \tag{A1}$$

where brackets represent ensemble averages. Express $\hat{C}_{12}(\tau)$ as a function of its expected value $C_{12}(\tau)$ plus a small zero mean perturbation $\Delta C_{12}(\tau)$, and use equations (A4) and (A5) from a previous paper [*Zhang et al.*, 2003] in above equation to obtain the following equation for the covariance of the cross-correlation magnitude estimates at lags τ_1, τ_2

$$\begin{aligned}\mathrm{cov}\left[\left|\hat{C}_{12}(\tau_1)\right|, \left|\hat{C}_{12}(\tau_2)\right|\right] = {} & |C_{12}(\tau_1)||C_{12}(\tau_2)| \\ & \times \frac{1}{4}\left[\frac{\langle\Delta C_{12}(\tau_1)\Delta C_{12}(\tau_2)\rangle}{C_{12}(\tau_1)C_{12}(\tau_2)} + \frac{\langle\Delta C^*_{12}(\tau_1)\Delta C^*_{12}(\tau_2)\rangle}{C^*_{12}(\tau_1)C^*_{12}(\tau_2)} \right. \\ & \left. + \frac{\langle\Delta C_{12}(\tau_1)\Delta C^*_{12}(\tau_2)\rangle}{C_{12}(\tau_1)C^*_{12}(\tau_2)} + \frac{\langle\Delta C^*_{12}(\tau_1)\Delta C_{12}(\tau_2)\rangle}{C^*_{12}(\tau_1)C_{12}(\tau_2)}\right].\end{aligned} \tag{A2}$$

[41] In the presence of noise, the signal voltage can be written as a sum of the echo voltage from the resolution volume and that voltage associated with internally generated noise,

$$E(t) = E^{(S)}(t) + E^{(N)}(t), \tag{A3}$$

where $E^{(S)}(t)$ is the resolution volume echo voltage or signal, and $E^{(N)}(t)$ is the noise voltage. A time series of signal samples from the two receivers are used to estimate the auto and cross-correlation functions using time averaging with a finite number of samples. Suppose the signals are measured at time step of T_s, and $\tau = nT_s$. Then the cross-correlation estimator is

$$\begin{aligned}\hat{C}_{12}(\tau) &= \frac{1}{M}\sum_{m=1}^{M} E_1(m)E_2^*(m+n) \\ &= \frac{1}{M}\sum_{m=1}^{M}\left[E_1^{(S)}(m) + E_1^{(N)}(m)\right] \\ &\quad \cdot \left[E_2^{(S)*}(m+n) + E_2^{(N)*}(m+n)\right]\end{aligned} \tag{A4}$$

Because the expected value of the cross-correlation function is given by (2), subtraction of the expected correlation function C_{12} from (A4) yields the estimation error ΔC_{12},

$$\begin{aligned}\Delta C_{12}(\tau) = {} & \frac{1}{M}\sum_{m=1}^{M}\left[E_1^{(S)}(m) + E_1^{(N)}(m)\right] \\ & \cdot \left[E_2^{(S)*}(m+n) + E_2^{(N)*}(m+n)\right] - C_{12}(n),\end{aligned} \tag{A5}$$

having statistics that can be developed as follows.

[42] The covariance of the cross-correlation estimates is $\langle \Delta C_{12}(\tau_1)\Delta C_{12}(\tau_2)\rangle$

$$= \left\langle \left\{ \frac{1}{M}\sum_{m=1}^{M}\left[E_1^{(S)}(m) + E_1^{(N)(m)}\right] \cdot \left[E_2^{(S)*}(m+n_1) + E_2^{(N)*}(m+n_1)\right] - C_{12}(n_1)\right\} \cdot \left\{ \frac{1}{M}\sum_{m'=1}^{M}\left[E_1^{(S)}(m') + E_1^{(N)}(m')\right] \cdot \left[E_2^{(S)*}(m'+n_2) + E_2^{(N)*}(m'+n_2)\right] - C_{12}(n_2)\right\}\right\rangle$$
$$= \frac{1}{M^2}\sum_{m=1}^{M}\sum_{m'=1}^{M}\left\langle \left[E_1^{(S)}(m) + E_1^{(N)}(m)\right] \cdot \left[E_2^{(S)*}(m+n_1) + E_2^{(N)*}(m+n_1)\right] \cdot \left[E_1^{(S)}(m') + E_1^{(N)(m')}\right]\left[E_2^{(S)*}(m'+n_2) + E_2^{(N)*} \cdot (m'+n_2)\right]\right\rangle - C_{12}(n_1)C_{12}(n_2). \quad \text{(A6)}$$

We assume that signal and noise, as well as noise from different receivers and at lags different than zero, are not correlated. That is:

$$\left\langle E_p^{(S)}(m)E_q^{(N)*}(m')\right\rangle = 0 \quad \text{(A7a)}$$

$$\left\langle E_p^{(N)}(m)E_q^{(N)*}(m')\right\rangle = N\delta(p-q)\delta(m-m'). \quad \text{(A7b)}$$

Noise samples at lags different than zero will be uncorrelated provided the receiver bandwidth is large compared to the sample spacing T_s, and noise from different receivers will be uncorrelated provided that the noise is internally generated in each of the receivers. Expressing the fourth moment of the zero-mean Gaussian random process in terms of the products of the second moments [*Ishimaru*, 1997, p. 450], (A6) can be written as

$$\langle \Delta C_{12}(\tau_1)\Delta C_{12}(\tau_2)\rangle = \frac{1}{M^2}\sum_{m=1}^{M}\sum_{m'=1}^{M} \cdot \left[\left\langle E_1^{(S)}(m)E_2^{(S)*}(m+n_1)E_1^{(S)}(m')E_2^{(S)*}(m'+n_2)\right\rangle + \left\langle E_1^{(S)}(m)E_2^{(S)*}(m+n_1)E_1^{(N)}(m')E_2^{(N)*}(m'+n_2)\right\rangle + \left\langle E_1^{(N)}(m)E_2^{(N)*}(m+n_1)E_1^{(S)}(m')E_2^{(S)*}(m'+n_2)\right\rangle + \left\langle E_1^{(N)}(m)E_2^{(N)*}(m+n_1)E_1^{(N)}(m')E_2^{(N)*}(m'+n_2)\right\rangle\right] - C_{12}(n_1)C_{12}(n_2)$$
$$= \frac{1}{M^2}\sum_{m=1}^{M}\sum_{m'=1}^{M}\left[\left\langle E_1^{(S)}(m)E_2^{(S)*}(m+n_1)\right\rangle\left\langle E_1^{(S)}(m') \cdot E_2^{(S)*}(m'+n_2)\right\rangle + \left\langle E_1^{(S)}(m)E_2^{(S)*}(m'+n_2)\right\rangle \cdot \left\langle E_1^{(S)}(m')E_2^{(S)*}(m+n_1)\right\rangle\right] - C_{12}(n_1)C_{12}(n_2)$$
$$= \frac{1}{M^2}\sum_{m=1}^{M}\sum_{m'=1}^{M} C_{12}^{(S)}(m'-m+n_2)C_{12}^{(S)}(m-m'+n_1)$$
$$= \frac{1}{M}\sum_{m=-(M-1)}^{M-1}\left(1-\frac{|m|}{M}\right)C_{12}^{(S)}(m+n_2)C_{12}^{(S)}(-m+n_1). \quad \text{(A8)}$$

[43] Note that noise power does not contribute to $\langle \Delta C_{12}(\tau_1)\Delta C_{12}(\tau_2)\rangle$. We assume the time step T_{se} is much shorter than correlation time (i.e., $T_{se} \ll \tau_c$), and a dwell time (i.e., the averaging time to make an estimate) $T_d = MT_{se}$ much longer than the correlation ($T_d \gg \tau_c$). Under these conditions, the summation in (A8) can be evaluated using the integral

$$\approx \frac{1}{T_d}\int C_{12}^{(S)}(t+\tau_1)C_{12}^{(S)}(-t+\tau_2)dt$$

to yield, for the assumed Gaussian form of the cross-correlation function given by (3),

$$= \frac{1}{M_I}C_{12}^{(S)}(\tau_1)C_{12}^{(S)}(\tau_2)\exp\left(\frac{(\tau_2-\tau_1)^2}{4\tau_c^2}\right), \quad \text{(A9)}$$

in which the number of independent signal samples in T_d is $M_I = \frac{T_d}{\sqrt{\pi}\tau_c}$.

[44] Similarly, we obtain

$$\langle \Delta C_{12}^*(\tau_1)\Delta C_{12}^*(\tau_2)\rangle = \frac{1}{M_I}C_{12}^{(S)*}(\tau_1)C_{12}^{(S)*}(\tau_2) \cdot \exp\left(\frac{(\tau_2-\tau_1)^2}{4\tau_c^2}\right). \quad \text{(A10)}$$

[45] Likewise, using the procedure leading to (A9), we have

$$\langle \Delta C_{12}(\tau_1)\Delta C_{12}^*(\tau_2)\rangle = \frac{1}{M^2}\sum_{m=1}^{M}\sum_{m'=1}^{M}\left[\left\langle E_1^{(S)}(m)E_2^{(S)*} \cdot (m+n_1)E_1^{(S)*}(m')E_2^{(S)}(m'+n_2)\right\rangle. + \left\langle E_1^{(S)}(m)E_2^{(N)*}(m+n_1)E_1^{(S)*}(m')E_2^{(N)}(m'+n_2)\right\rangle\right.$$

$$
\begin{aligned}
&+\left\langle E_1^{(N)}(m)E_2^{(S)*}(m+n_1)E_1^{(N)*}(m')E_2^{(S)}(m'+n_2)\right\rangle \\
&+\left\langle E_1^{(N)}(m)E_2^{(N)*}(m+n_1)E_1^{(N)*}(m')E_2^{(N)}(m'+n_2)\right\rangle\Big] \\
&- C_{12}(n_1)C_{12}^*(n_2) \\
=&\frac{1}{M^2}\sum_{m=1}^{M}\sum_{m'=1}^{M}\Big[\left\langle E_1^{(S)}(m)E_2^{(S)*}(m+n_1)\right\rangle \\
&\cdot\left\langle E_1^{(S)*}(m')E_2^{(S)}(m'+n_2)\right\rangle \\
&+\left\langle E_1^{(S)}(m)E_1^{(S)*}(m')\right\rangle\left\langle E_2^{(S)}(m'+n_2)E_2^{(S)*}(m+n_1)\right\rangle \\
&+\left\langle E_1^{(S)}(m)E_1^{(S)*}(m')\right\rangle\left\langle E_2^{(N)}(m'+n_2)E_2^{(N)*}(m+n_1)\right\rangle \\
&+\left\langle E_1^{(N)}(m)E_1^{(N)*}(m')\right\rangle\left\langle E_2^{(S)}(m'+n_2)E_2^{(S)*}(m+n_1)\right\rangle \\
&+\left\langle E_1^{(N)}(m)E_2^{(N)*}(m')\right\rangle\left\langle E_1^{(N)}(m'+n_2)E_2^{(N)*}(m+n_1)\right\rangle\Big] \\
&-C_{12}(n_1)C_{12}(n_2) \\
=&\frac{1}{M^2}\sum_{m=1}^{M}\sum_{m'=1}^{M}\Big[C_{11}^{(S)}(m'-m)C_{11}^{(S)}(m+n_1-m'-n_2) \\
&+C_{11}^{(S)}(m'-m)N\delta(m+n_1-m'-n_2) \\
&+N\delta(m'-m)C_{11}^{(S)}(m+n_1-m'-n_2) \\
&+N\delta(m'-m)N\delta(m+n_1-m'-n_2) \\
=&\frac{1}{M}\sum_{m=-(M-1)}^{M-1}\left(1-\frac{|m|}{M}\right)C_{11}^{(S)}(m)C_{11}^{(S)}(-m+n_1-n_2) \\
&+\frac{2}{M}NC_{11}^{(S)}(n_1-n_2)+\frac{1}{M}N^2\delta(n_1-n_2) \\
\approx&\frac{1}{T_d}\int C_{11}^{(S)}(t)C_{11}^{(S)}(-t+\tau_1-\tau_2)dt+\frac{2N}{M}C_{11}^{(S)}(\tau_1-\tau_2) \\
&+\frac{N^2}{M}\delta(\tau_1-\tau_2).
\end{aligned}
$$

[46] For the Gaussian functional form of the auto-correlation function given by (2), the integral can be evaluated to yield

$$
\begin{aligned}
\langle\Delta C_{12}(\tau_1)\Delta C_{12}^*(\tau_2)\rangle =& \frac{S^2}{M_I}\exp(-jw_d(\tau_1-\tau_2)) \\
&\cdot\exp\left(-\frac{(\tau_1-\tau_2)^2}{4\tau_c^2}\right)+\frac{2N}{M}C_{11}^{(S)}(\tau_1-\tau_2) \\
&+\frac{N^2}{M}\delta(\tau_1-\tau_2). \qquad \text{(A11)}
\end{aligned}
$$

[47] Similarly, we have

$$
\begin{aligned}
\langle\Delta C_{12}^*(\tau_1)\Delta C_{12}(\tau_2)\rangle =& \frac{S^2}{M_I}\exp(-j\omega_d(\tau_2-\tau_1)) \\
&\cdot\exp\left(-\frac{(\tau_2-\tau_1)^2}{4\tau_c^2}\right)+\frac{2N}{M}C_{11}^{(S)}(\tau_2-\tau_1) \\
&+\frac{N^2}{M}\delta(\tau_2-\tau_1). \qquad \text{(A12)}
\end{aligned}
$$

[48] Substituting (A9) to (A12) into (A2) yields the following expression for the covariance of the magnitudes of cross-correlation function at lags τ_1 and τ_2:

$$
\begin{aligned}
\mathrm{cov}\left[|\hat{C}_{12}(\tau_1)|,|\hat{C}_{12}(\tau_2)|\right] =& \frac{1}{2M_I}\left(S^2\exp\left(-\frac{(\tau_2-\tau_1)^2}{4\tau_c^2}\right)\right. \\
&\left.+|C_{12}^{(S)}(\tau_1)||C_{12}^{(S)}(\tau_2)|\exp\left(\frac{(\tau_2-\tau_1)^2}{4\tau_c^2}\right)\right) \\
&+\frac{N}{M}|C_{11}^{(S)}(\tau_1-\tau_2)|+\frac{N^2}{2M}\delta(\tau_1-\tau_2). \qquad \text{(A13)}
\end{aligned}
$$

If $\tau_1 = \tau_2$, (A13) reduces to the variance of the estimates of the cross-correlation magnitude. Thus

$$
\mathrm{var}\left[|\hat{C}_{12}(\tau)|\right] = \frac{1}{2M_I}\left(S^2+|C_{12}^{(S)}(\tau)|^2\right)+\frac{SN}{M}+\frac{N^2}{2M}. \qquad \text{(A14)}
$$

[49] Using the same procedure, we obtain the following expression for the variance of auto-correlation function:

$$
\begin{aligned}
\mathrm{var}\left[|\hat{C}_{11}(\tau)|\right] =& \frac{1}{2M_I}\left(S^2+|C_{11}^{(S)}(\tau)|^2\right)+\frac{N}{M} \\
&\times\left(S+|C_{11}^{(S)}(2\tau)|\right)+\frac{N^2}{2M}[1+\delta(n)], \qquad \text{(A15)}
\end{aligned}
$$

under the condition $|\tau| \ll T_d = MT_s$. Likewise the covariance of the auto- and cross-correlation functions is given by,

$$
\begin{aligned}
\mathrm{cov}\left[|\hat{C}_{11}(\tau_1)||\hat{C}_{12}(\tau_2)|\right] =& \frac{1}{2M_I}|C_{11}^{(S)}(\tau_1)||C_{12}^{(S)}(\tau_2)| \\
&\cdot\left(\exp\left(\frac{(\tau_2-\tau_1-\tau_p)^2}{4\tau_c^2}\right)+\exp\left(\frac{(\tau_2+\tau_1-\tau_p)^2}{4\tau_c^2}\right)\right) \\
&+\frac{N}{2M}\left(|C_{12}^{(S)}(\tau_1+\tau_2)|+|C_{12}^{(S)}(\tau_2-\tau_1)|\right). \qquad \text{(A16)}
\end{aligned}
$$

Equations (A13)–(A16) constitute the variance and covariance for the magnitude of auto- and cross-correlation estimates in the presence of noise. The standard error of the baseline wind estimates can be derived from them.

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