Efficient Atmospheric Simulation for High-Resolution Radar Imaging Applications

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ABSTRACT

Numerical simulation can be used for optimizing radar imaging techniques because it allows the accuracy of various techniques to be studied. A simulation of atmospheric conditions by using scatterers in a 3D volume was proposed by Holdsworth and Reid. For this method, the computational burden increases as the square of the number of scatterers. Hence, the simulation can become time consuming if a large number of scatterers is required in order to simulate atmospheric conditions more realistically. A method based on table lookup and linear interpolation is proposed to replace the existing turbulent wind field generation of Holdsworth and Reid. Moreover, this method has the flexibility of incorporating other turbulent wind field data, either measured or modeled. Pseudocode of the simulation using this method is included, and a comparison of the computational burden for each of the techniques is presented.

1. Introduction

The optimization of atmospheric radar processing techniques can be accomplished by using numerical simulations. By having complete knowledge of the simulated atmospheric conditions, the accuracy of various techniques in estimating atmospheric parameters can be studied. A flexible radar simulation method using many scattering points in a 3D volume was proposed by Holdsworth and Reid (1995). This type of radar simulation has been extensively used and is suitable for many types of atmospheric studies (e.g., Yu 2000).

Due to computational constraints, the number of scatterers for this technique is typically limited to 200–300. However, several thousand scatterers were deemed necessary for the proper simulation of volume scattering with a higher-resolution imaging system such as the Turbulent Eddy Profiler (TEP). Developed at the University of Massachusetts Amherst (Mead et al. 1998), TEP uses over 60 independent receivers for imaging of the lower atmosphere. Unfortunately, the method developed by Holdsworth and Reid (1995) uses a 3D spatial filter to simulate spatially correlated turbulent wind fields from a random field; this filter has a large computational burden that precludes its use with thousands of scattering points. A more computationally efficient method for incorporating spatially correlated turbulent wind fields is proposed that uses table lookup and linear interpolation to update the scatterers' wind vectors. In addition to making the simulation computationally feasible, this method allows straightforward incorporation of data from any turbulent field, simulated or measured.

2. Overview and motivation

The simulation method of Holdsworth and Reid (1995) is initiated by randomly positioning a finite number of scatterers inside an enclosing volume. At each sampling time, the total received signal is simply the complex sum of all the returned echoes from the scatterers. Each returned echo is a function of the random scatterer reflectivity, the antenna beam pattern, the aspect sensitivity, the range weighting function, and the two-way path distance from the transmitter to the scatterer. The positions of the scatterers are updated for each sampling time according to the effects of the mean wind field and spatially correlated turbulent air movement. When a scatterer moves outside the enclosing volume, it is replaced by a new scatterer on the opposite side of the enclosing volume. A scaled white noise sequence can be added to the composite returned signal at each receiver in order to achieve a desired signal-to-noise ratio (SNR).

As mentioned earlier, the turbulent wind field is generated by spatially filtering (or smoothing) random turbulent velocities associated with each scatterer. This update is typically done for each scatterer at each sampling time and involves computing the effect of each scatterer in the volume on every other scatterer. A problem arises when the number of scatterers is increased. For example, if 200 scatterers are placed in the enclosing volume, there are at least $(2 \times 200)^2$ computations (multipli-

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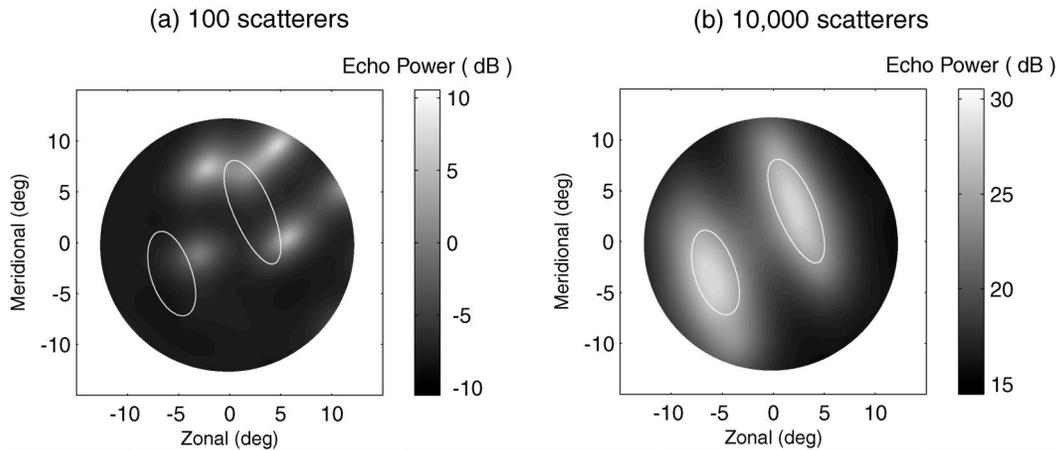E-mail: boonleng@doppler.unl.edu

(a) 100 scatterers (b) 10,000 scatterers

FIG. 1. Simulated echo power images for SNR = 3 dB using (a) 100 and (b) 10 000 scatterers. The white contour lines represent the −6 dB returned echo from the reflectivity model. Using the TEP configuration, individual and superposition of scatterers are imaged in (a), exemplifying the need for a higher number of scatterers to realistically simulate volume scattering. (b) The imaging result with 10 000 scatterers.

cations and additions) to be made for each time sample. If the desired number of scatterers is increased to 10 000, for example, one can easily see that the computational burden is increased to $(2 \times 10\,000)^2$. Therefore, a simulation using thousands of scatterers becomes intractable. The computational burden could be decreased by not filtering the turbulent field at each time step or over the entire enclosing volume. However, adverse effects can be manifested in the spatial/temporal continuity of the data. If spatial/temporal continuity is omitted by eliminating either the spatial or temporal filter in the generation of the turbulent velocity field, individual scatterers could have abrupt changes in turbulent velocity as a function of position or time, respectively. It is mentioned in Holdsworth and Reid (1995) that abrupt changes in the turbulent velocity result in "spikes" at zero lag in the autocorrelation function. This is undesirable in radar imaging applications that depend on the zero-lag autocorrelation function.

The impetus for this development was the realistic simulation of the TEP radar system. During preliminary studies using the original method, it was discovered that a larger number of scatterers were necessary for reasonable results. The data presented in this study will use a TEP array configuration with 61 receivers. The angular resolution of the simulated TEP array is higher than previous studies using fewer than 10 receivers (e.g., Yu 2000). For a relatively modest number of scatterers, say fewer than 200, imaging using the TEP array is capable of distinguishing individual scatterers in the volume of interest. Figure 1 illustrates a comparison of the echo power maps with 100 and 10 000 scatterers in the simulation. The model reflectivity field consists of two relatively wide bivariate Gaussian functions centered at (2°, 4°) and (−6°, −4°). Both maps are produced using Capon's method of beam forming (Palmer et al. 1998). Note that distinct maxima in the map are visible

for the 100-scatterer case, although volume scattering was desired. Each of the maxima could correspond to a single scatterer or a combination of multiple scatterers. Since the simulation is done using a larger enclosing volume, approximately 15% of the scatterers did not pass through the imaging region. Furthermore, the weighting function of the reflectivity model can cause scatterers to have lower returned power in the image.

In order to model the atmospheric scatter more realistically for the TEP array, a larger number of scatterers is needed. Thus, the proposed numerical simulation method is motivated by the desired volume scattering simulation.

3. Simulation overview

a. Basic structure of simulation

A large number of scatterers in an enclosing volume is first initialized with a random reflectivity. At each sampling time, the total returned power for each scatterer is calculated as a product of the reflectivity model, the range weighting function, and the beam-pattern weighting function. The total reflectivity for a given position (x, y, z) is the product of horizontal and vertical reflectivity, $w_{mh}(x, y)$ and $w_{mv}(z)$, respectively. They are computed separately, as a bivariate Gaussian function (horizontal) and an independent single-variate Gaussian function (vertical), as follows (Holdsworth and Reid 1995):

$$w_{mh}(x, y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1 - \rho^2}} \exp\left(-\frac{\alpha}{2\sqrt{1 - \rho^2}}\right) \quad (1)$$

$$w_{mv}(z) = \frac{1}{\sqrt{2\pi}\sigma_z} \exp\left[-\frac{(z - \mu_z)^2}{\sigma_z^2}\right] \quad (2)$$
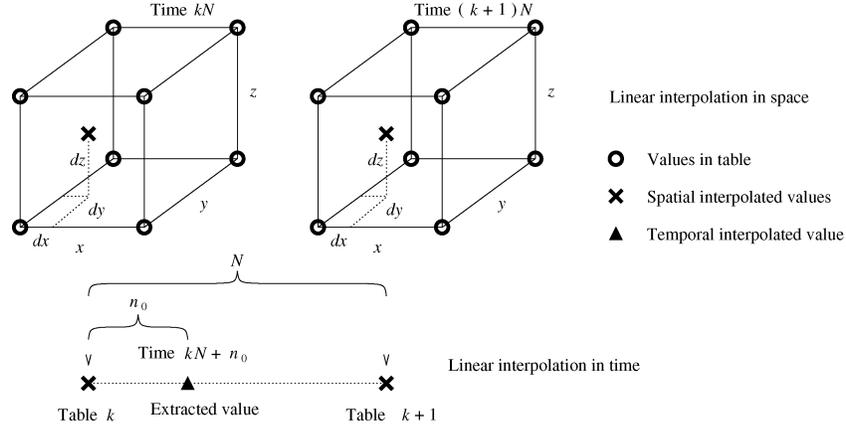
where

FIG. 2. At time $(kN + n_0)$, two linearly interpolated values in space are extracted from tables k and $(k + 1)$, then these two values are linearly interpolated in time to obtain the final turbulent value.

$$\alpha = \frac{(x - \mu_x)^2}{\sigma_x^2} + \frac{(y - \mu_y)^2}{\sigma_y^2} - \frac{2\rho(x - \mu_x)(y - \mu_y)}{\sigma_x \sigma_y}$$

$$\rho \triangleq \frac{\sigma_{xy}}{\sigma_x \sigma_y},$$

where σ_x, σ_y, and σ_z are the standard deviations for the x, y, and z directions, respectively, and μ_x, μ_y, and μ_z are the means for the x, y, and z directions, respectively. Finally, ρ is defined as the correlation coefficient of x and y for the bivariate Gaussian function.

The range weighting function is applied using the following equation (Holdsworth and Reid 1995; Yu 2000):

$$w_r(x, y, z) = \exp\left[-\frac{(r - \mu_r)^2}{4\sigma_r^2}\right], \qquad (3)$$

where $r = \sqrt{x^2 + y^2 + z^2}$ is the range, and the variance σ_r is typically given by $\sigma_r = 0.35c\tau/2$, where c is the speed of light and τ is the pulse width (Doviak and Zrnić 1993).

Finally, the beam-pattern weighting function is given by (Yu 2000):

$$w_b(x, y) = \exp\left[-\frac{(\theta_x - \overline{\theta}_x)^2}{2\overline{\sigma}_x^2} - \frac{(\theta_y - \overline{\theta}_y)^2}{2\overline{\sigma}_y^2}\right] \qquad (4)$$

where

$$\theta_x = \tan^{-1}\left(\frac{x}{z}\right) \qquad \theta_y = \tan^{-1}\left(\frac{y}{z}\right),$$

where $\overline{\theta}_x$ and $\overline{\theta}_y$ describe the antenna beam-pointing direction in degrees, and $\overline{\sigma}_x$ and $\overline{\sigma}_y$ describe the beam-width in degrees in the x and y directions, respectively.

The phase of the returned signal is simply a function of the two-way distance traveled D and the wavelength λ. The in-phase and quadrature components of the re-turned signal are then computed as the real and imaginary components of the composite signal

$$x(t) = \sum_{k=0}^{N-1} A^{(k)} \exp\left[-j\frac{2\pi D^{(k)}}{\lambda}\right], \qquad (5)$$

where N is the total number of scatterers and $A^{(k)}$ is the amplitude of the returned signal, which is the square root of the product of the weighting functions, described as

$$A^{(k)} = \sqrt{w_{\text{mh}}^{(k)} w_{\text{mv}}^{(k)} w_r^{(k)} w_b^{(k)}}.$$

As time evolves, scatterer positions are updated based on the mean and turbulent velocity fields, which are discussed in the next section.

b. Turbulent velocity update procedure

Pregenerated turbulent velocity fields over a volume are stored in a tabulated format. The turbulent field could originate from fluid-dynamics-based simulations, such as direct numerical simulation (DNS), or from actual measurements. The turbulent field source is unrelated to the velocity update procedure.

The tabulated values are initially assigned on a uniform 3D grid within the enclosing volume. For any time instance, two tables are used to simulate the temporal continuity in turbulent velocity changes. Figure 2 illustrates the procedure of extracting the turbulent velocity for an example of time $(kN + n_0)$ at position (x, y, z). The nearest eight values per table (denoted by circles in the figure) are extracted from tables k and $(k + 1)$. Then these two sets of eight values are linearly interpolated in space in order to get two turbulent velocities at the position (x, y, z). Lastly, these two values are linearly interpolated in time to get the turbulent velocity at position (x, y, z) for turbulence at time $(kN + n_0)$.

Linear interpolation can be simply described as follows:

$$f_i(x) = f(\lfloor x \rfloor) + \frac{x - \lfloor x \rfloor}{\lceil x \rceil - \lfloor x \rfloor}[f(\lceil x \rceil) - f(\lfloor x \rfloor)], \quad (6)$$

where $\lceil x \rceil$ is the smallest integer larger than x and $\lfloor x \rfloor$ is the largest integer smaller than x.

An artifact of linear interpolation is a systematic change in the variance of the turbulent wind field. By treating the turbulence as a random variable X, one can observe that the intermediate value is described as

$$X_\beta = (1 - \beta)X_1 + \beta X_2, \quad (7)$$

where β is the intermediate value taken from 0 to 1, X_1 is the initial value, and X_2 is the final value. Suppose $\text{VAR}(X_1) = \text{VAR}(X_2) = \text{VAR}(X)$: a direct calculation shows that the variance of the interpolated random variable X_β is

$$\text{VAR}(X_\beta) = [(1 - \beta)^2 + 2\rho(1 - \beta)\beta + \beta^2] \, \text{VAR}(X).$$
$$(8)$$

This causes $\text{VAR}(X_\beta)$ to be reduced in a quadratic form as a function of β as seen in Eq. (8). In order to reverse this artifact, the interpolated value can be scaled with an inverse-square-root function of Eq. (8). So, the scaled interpolated value is

$$f_i'(x) = \frac{1}{\sqrt{(1 - \beta)^2 + 2\rho(1 - \beta)\beta + \beta^2}} f_i(x), \quad (9)$$

where ρ is the cross-correlation factor of X_1 and X_2. The solution in Eq. (9) is incorporated into the linear interpolation by using $\beta = n_0/N$.

The proposed table lookup method is fast and efficient. This modification reduces the computational burden substantially from the conventional spatially correlated random turbulent model (Holdsworth and Reid 1995). For 10 000 scatterers, the table lookup method requires approximately 75 floating point operations (FLOPS) to compute the turbulent velocity update for each scatterer, whereas the conventional spatial-correlation method requires at least 20 000 FLOPS per scatterer. As a result, the proposed method allows high-resolution radar simulation with a realistic turbulent velocity field using thousands of scattering points.

4. Pseudocode

In order to facilitate the use of this efficient algorithm, the following pseudocode is provided for the case of multiple receiver simulations.

```
Assign random reflectivity power of each scatterer to vector RP
Loop (IDX_SCATTER → Number of scatterers)
 {
 RP(IDX_SCATTER) = 0.5*RAND + 0.5
 }
Loop (IDX_CH → Number of channels)
 {
 Loop (IDX_PT → Number of points)
  {
  TOT_POWER = 0;
  Loop (IDX_SCATTER → Number of scatterers)
   {
   Calculate two way path distance, DISTANCE
   Compute horizontal model weight, W_MH
   Compute the vertical model weight, W_MV
   Compute range weight, W_R
   Compute beampattern-weight, W_B
   TOT_POWER = W_MH × W_MV × W_B × W_R × RP(IDX_SCATTER)
   AMP = √TOT_POWER
   IN_PH(IDX_CH)(IDX_PT) ← IN_PH(IDX_CH)(IDX_PT) + AMP × cos(−2π DISTANCE/λ)
   QUADT(IDX_CH)(IDX_PT) ← QUADT(IDX_CH)(IDX_PT) + AMP × sin(−2π DISTANCE/λ)
   }
  }
 Loop (IDX_SCT → Number of scatterers)
  {
  Extract closest 8 turbulent values at position (x, y, z) from OLD_TURB_TABLE
  Extract closest 8 turbulent values at position (x, y, z) from NEW_TURB_TABLE
  Linearly interpolate in space to obtain the wind field for each table at (x, y, z)
  Linearly interpolate these 2 turbulent field values in time.
  Compute the scaling factor to undo the variance change in interpolation
```

```
TURB(x, y, z) → VAR_SCALE × TURB(x, y, z)
TOT_WIND = MEAN_VELOCITY + TURB(x, y, z)
Use TOT_WIND to compute the next possible position
If (next possible position is outside the enclosing volume)
  {
  Replace the scatterer with a new scatterer in the opposite location
  }
Update new position of the scatterer
  }
 }
}
```

5. Conclusions

The primary contribution of this paper is the development of a computationally efficient method for simulating atmospheric conditions for multiple receiver radars. The method can incorporate either actual or synthetic turbulent fields. Pseudocode is included for an atmospheric scatterer model that can incorporate either modeled or measured turbulent fields in a more computationally efficient manner than the existing Holdsworth and Reid (1995) technique. This allows the simulation to be effective for high-resolution profiling systems.

Mathematical expressions of the critical parameters for the simulation are included, and a comparison is made between the computational requirements of the spatially correlated random turbulent velocity generator and the proposed table lookup method. The table lookup scheme is recommended for use in simulations of the atmosphere, especially when a large number of scatterers is desired or required. The technique also presents a convenient way of incorporating a variety of modeled or measured turbulent data.

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