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THE COEFFICIENT OF KINETIC FRICTION FOR ALUMINUM

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Summary

The translational and rotational motions of a cylinder traveling down a straight incline, which provides a constant linear acceleration, depend on the angle of inclination. For small angles above the horizontal, free rolling takes place, but, above a critical angle, rolling and slipping occur. This critical angle is used to evaluate the coefficient of friction for steel cylinders progressing over aluminum surfaces. The moment of inertia is varied by using solid and annular cylinders and the aluminum surface is smooth, rough or corrugated. The coefficient of friction is shown to be independent of the moment of inertia but to depend on the type of surface and the angle of inclination.

1. Introduction

The study of mechanics readily describes the translational and rotational motions of spheres and cylinders on a plane surface [1]. However, experimental observations of the simultaneous actions of rolling and sliding motions for spheres are only now being published [2, 3]. To our knowledge, nothing has been published which describes the similar actions for cylinders. This type of problem, however, is quite important and has many practical applications. For example, before one can apply either the adhesion or the delamination theory of wear [4] one must have information about how much seizure and slipping occur between two interacting surfaces.

Our prior studies [3] of the rolling and sliding motions of spheres were made by using a computer to measure short time intervals for travel over known distances for a track inclined at definite angles. These data were used to evaluate the linear acceleration as a function of the angle of the track. The same procedure has been modified to increase the accuracy of the measurements so as to evaluate the natural motions of cylinders. For spheres or cylinders traveling on a plane surface, the horizontal position of the plane constitutes a case of free rolling while the vertical orientation corresponds to free fall. Accordingly the study of motion over a plane whose

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angle of inclination changes from 0° to 90° from the horizontal provides observations of all possible combinations of rolling and sliding motions.

First, we review the mathematical description for the rolling motions of cylinders and describe how the acceleration can be used to compute a value for the coefficient of kinetic friction. One advantage of using cylindrical objects is that the moment of inertia is readily altered by using solid or annular cylinders, but the same type of surface contact can be maintained by keeping a fixed outside radius and similar polished surfaces. The experimental data deal with the accelerations of steel cylinders whose moment of inertia is varied for motion over several types of plane surfaces of aluminum. Finally, a discussion is given about the way the coefficient of friction depends on the finish of an aluminum surface and on the angle of inclination.

2. Theory

Let us consider a right circular cylinder of mass m which is free to roll down an inclined plane while the axis of the cylinder maintains a horizontal orientation. Let the angle of inclination β of the plane be small so that there is no slippage between the cylinder and the flat inclined surface. Also let us assume that rolling friction is negligible so that the free-rolling motion is described mathematically by the torque equation (see for example ref. 5)

$$\tau = I\alpha \tag{1}$$

where I is the moment of inertia and α is the angular acceleration. By taking the torques about the point of contact between the cylinder and the plane eqn. (1) becomes

$$(mg \sin \beta)r = m(k^2 + r^2)\frac{a_r}{r} \tag{2}$$

where r is the radius of the cylinder, k is the radius of gyration, g is the gravitational acceleration and a_r is the linear acceleration for the free-rolling cylinder. Equation (2) is then solved to obtain the acceleration of the cylinder for free-rolling motion down an incline, *i.e.*

$$a_r = \frac{r^2}{k^2 + r^2} g \sin \beta \tag{3}$$

Whenever slipping occurs as the cylinder progresses down the incline the acceleration can be determined by using Newton's second law, *i.e.*

$$\Sigma F_i = ma_s \tag{4}$$

where the vector addition of the individual forces governs the acceleration a_s when slipping occurs. Again [5], resolving the weight of the cylinder into components acting parallel and perpendicular to the incline, we obtain the acceleration down the incline as

$$a_s = g(\sin\beta - \mu\cos\beta) \quad (5)$$

where m has been cancelled from each term and μ denotes the coefficient of kinetic friction.

One can define β_0 as that angle when the free-rolling motion is altered into a rolling and slipping behavior. This critical angle is readily determined by equating a_r and a_s, and when this is done, using eqns. (3) and (5), one can solve for a unique value of the coefficient of friction, *i.e.*

$$\mu_0 = \left(1 - \frac{r^2}{k^2 + r^2}\right) \tan\beta_0 \quad (6)$$

It should be noted that the precision with which one can evaluate the coefficient of kinetic friction is dependent on the measurements of the geometrical properties of the cylinder and the ability to evaluate β_0.

3. Experimental details

Solid and annular right circular cylinders of stainless steel were used. All the surfaces of the cylinders are highly polished and remain smooth and clean as a result of repeated cleaning with alcohol. The main geometrical properties of these cylinders are listed in Table 1. All the data in the table refer to cylinders with a length of 0.077 m. Some tests were made in which the length of the solid cylinders was varied but, as expected from theory, the results are independent of the cylinder length.

All the necessary experimental equipment is shown in Fig. 1. The inclined plane located in the center of the picture consists of a flat aluminum surface 0.5 m long. A metal frame holds the aluminum surface firmly and provides a hinged support to a vertical ringstand so that the angle of inclination of the plane is easily varied. The initial experiments were performed when the surface of the aluminum was polished. To determine the effect of a surface with a coarse texture, the flat aluminum was used after being ground with carborundum of grade 100 grit. Finally, experiments

TABLE 1

Physical properties of stainless steel cylinders with an outside radius of 1.27×10^{-2} m and a length of 7.7×10^{-2} m

Cylinder	*Mass* (kg)	*Inside radius* ($\times 10^{-3}$ m)
Solid	0.304	—
Annular 1	0.206	4.77
Annular 2	0.211	6.9
Annular 3	0.124	9.75
Annular 4	0.071	11.1

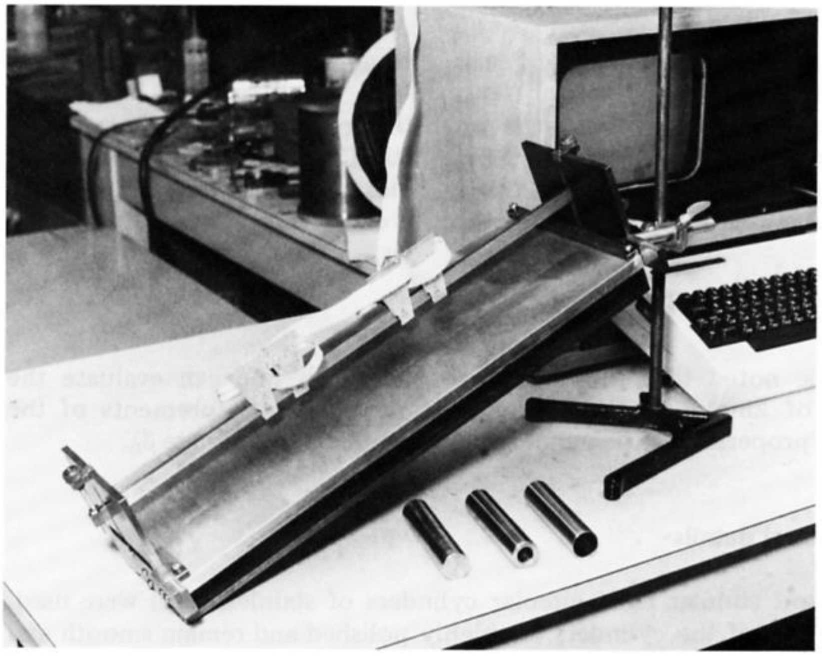


Fig. 1. The flat aluminum surface under investigation is shown positioned on a frame supported at one end by a ringstand. Detectors which respond to light reflected from a rolling cylinder are mounted slightly above the path traveled by the cylinder, and the electrical response is registered by an associated computer.

were performed after the aluminum had been machined with deep grooves so as to produce a corrugated or knurled finish. An evaluation of roughness for the three types of surfaces studied resulted in the approximate values tabulated in Table 2. However, because of the natural wear of asperities as successive sliding events took place, these numerical quantities were subject to considerable variations as an experiment progressed.

The measurements to evaluate the acceleration of a moving cylinder were made using two reflective switches (Spectronics Inc., type SPX 1404). The switches successively sense the moving object to give the time interval Δt for traveling the space between the switches, and these times were determined by and then stored in a computer. One switch was located at a fixed position on the inclined plane while the second switch was at a variable distance s from the initial position of the cylinder. For each location s of the switch three values of Δt were obtained and then a different s value was selected until a total of four s positions had been sampled. Values of Δt *versus* $s^{1/2}$ were used to compute the constant coefficients in the equation

$$\Delta t = C_0 s^{1/2} + C_1 \tag{7}$$

and then the acceleration was determined by using

$$a = 2C_0^{-2}$$

TABLE 2

Physical dimensions of the asperities for the various types of aluminum surfaces

	Average height of asperities (×10^{-5} m)	*Average distance between asperities* (×10^{-4} m)
Smooth surface	1	0.07
Rough surface	3	1
Corrugated surface	12	6

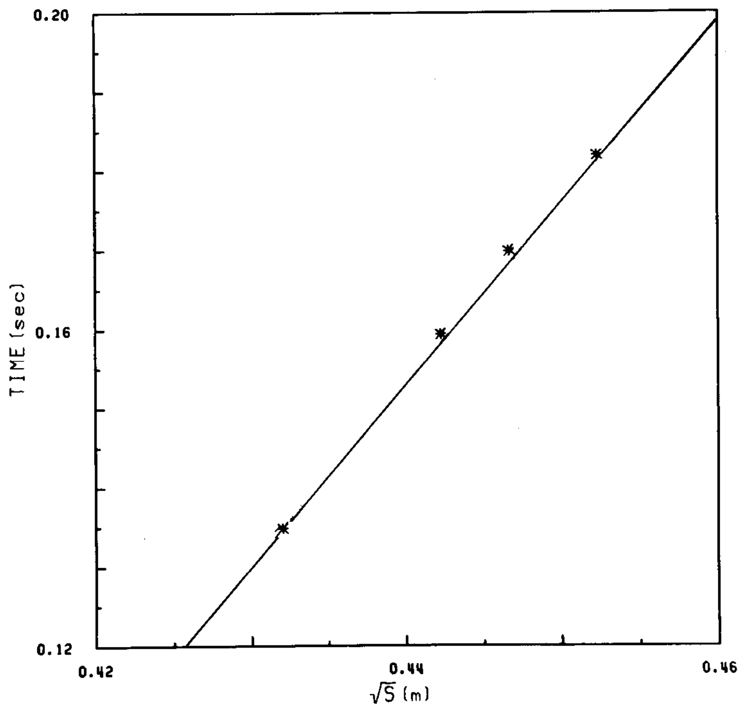


Fig. 2. The average time interval (✱) for a cylinder progressing a specific distance. ——, calculated from the experimental data. The slope of the line is used to compute the linear acceleration which corresponds to a particular angle of the inclined plane.

A typical result is shown in Fig. 2 where the slope $(2/a)^{1/2}$ of the data is known to a precision of ± 2%.

Typical results of these evaluations of the acceleration are presented in Fig. 3 for the case of a solid cylinder progressing over the smooth aluminum surface. The ordinate of the graph refers to the sine of the angle of inclination of the aluminum surface above the horizontal plane. For values of sin $\beta < 0.5$ the experimental data coincide with the full line which denotes free rolling. Values of acceleration deviate from that of the free-rolling motion when sin $\beta > 0.53$, and the data for these larger angles agree with the rolling and slipping action predicted by eqn. (5) which is shown by a broken line for $\mu = 0.2$.

In Fig. 3 the change from free rolling to rolling plus slipping is quite clearly shown by the discontinuity in the plot of the experimental data of acceleration *versus* sin β. It is this inflection which identifies the critical angle β_0.

Several annular cylinders were tested on the smooth aluminum surface, and the results for one of these are given in Fig. 4. The full and broken lines again correspond to the expected theoretical behavior. It should be noted that the major effect of reducing the moment of inertia is to reduce the slope of the line designating the free-rolling motion. Thus the critical angle for the slipping behavior is decreased when the moment of inertia is reduced, in agreement with eqn. (6).

To determine how the nature of the surface of the inclined plane influences the slipping action, the above experiments were repeated after the aluminum surface had been ground with grade 100 carborundum grit. As shown in Figs. 5 and 6, which correspond to the results of the solid and

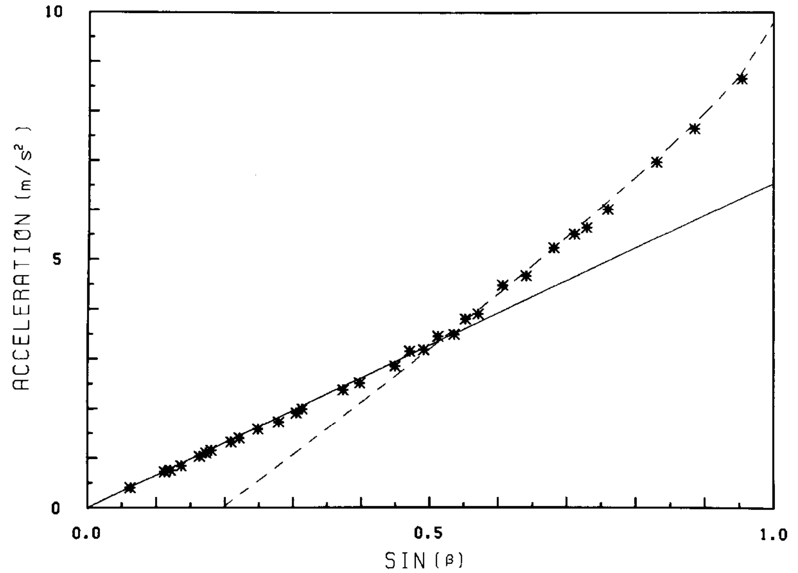


Fig. 3. Values (✱) for the linear acceleration of a solid cylinder on a smooth inclined plane ($k^2 = 81.0 \times 10^{-6}$ m^2). The data agree with the theoretical results determined for free rolling (———) and rolling and slipping (— — —).

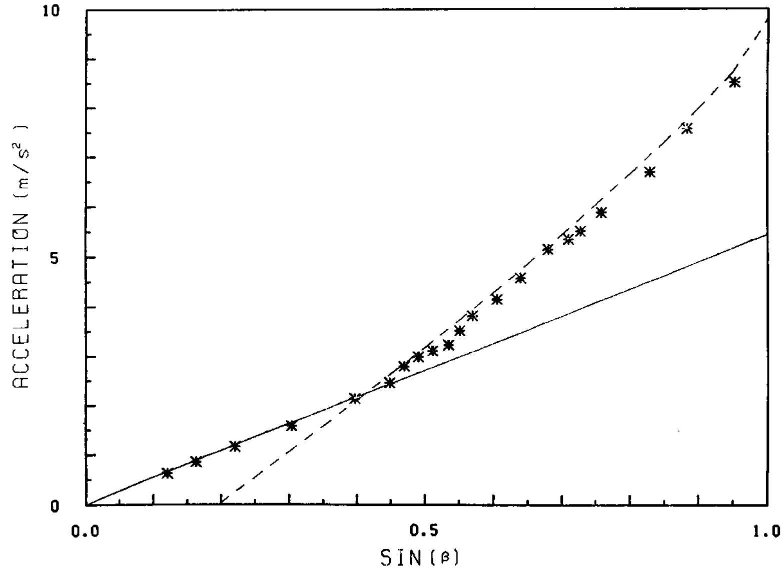


Fig. 4. The linear acceleration of an annular cylinder moving on a smooth surface is shown ($k^2 = 1.284 \times 10^{-4}$ m^2). There is good agreement between experiment and theory: ——, free rolling (theory); — — —, rolling and slipping (theory); ✱, experimental data.

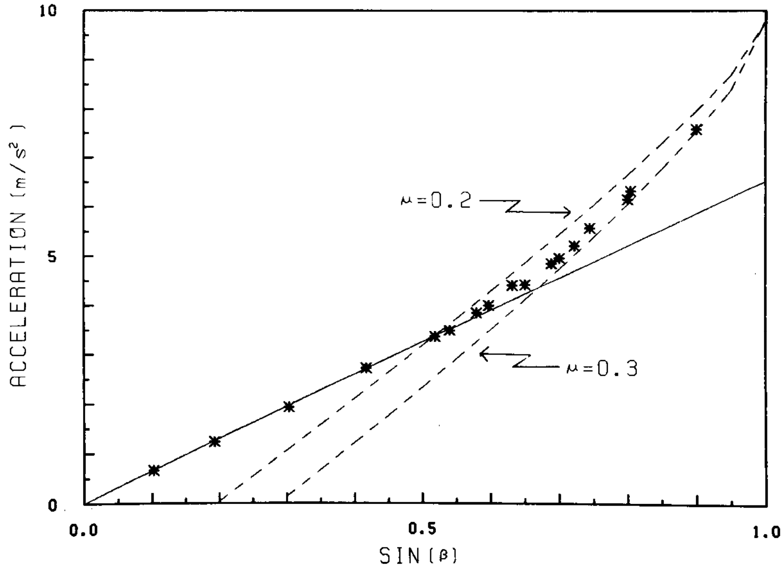


Fig. 5. A solid cylinder progressing over a rough surface ($k^2 = 81.0 \times 10^{-6}$ m^2) gives good agreement with the theoretical free-rolling motion (——). The rolling and slipping action of the cylinder does not agree with the theoretical data (— — —) determined by using a constant coefficient of friction.

an annular cylinder respectively, the free-rolling behavior does not depend on the aluminum surface, but there is a pronounced increase in friction for the slipping motion on the rough surface. When the angles of the inclined

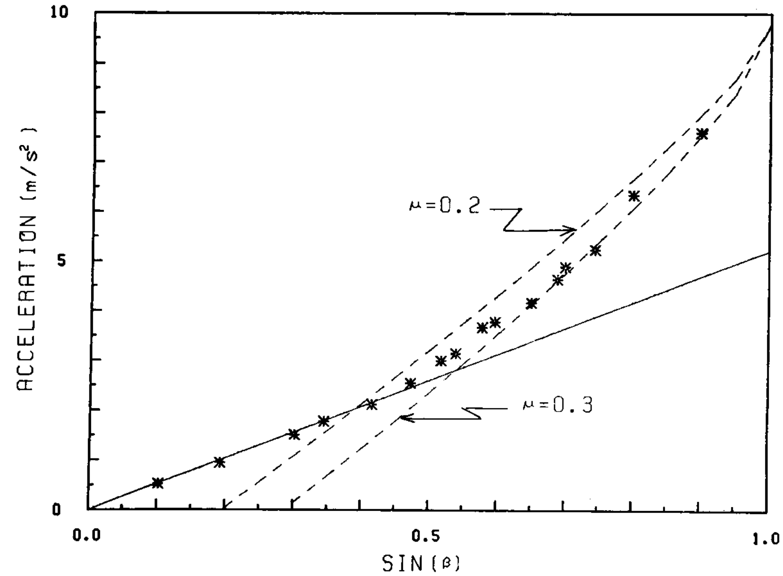
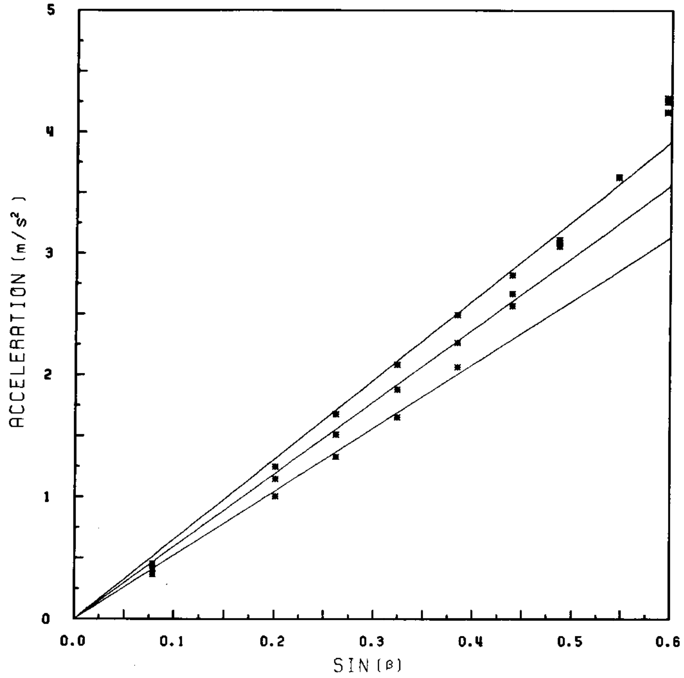


Fig. 6. The experimental points were determined with an annular cylinder progressing over a rough surface and the data are compared with the theoretical results ($k^2 = 1.426 \times 10^{-4}$ m^2): ✳, experimental data; ——, free rolling; — — —, rolling and slipping for constant values of the coefficient of friction.

plane exceed β_0, the values of the acceleration do not fit any single theoretical line for a constant μ value. Also, there seems to be an excessive amount of scatter in the data for these large angles. Since it became visibly apparent that the portion of the inclined plane over which the cylinders traveled was altered by its becoming polished during these measurements, we attribute the scatter in the values of the acceleration to wear of the asperities on the aluminum surface.

To confirm the fact that the coefficient of friction varies as the amount of slippage changes, an attempt was made to obtain a plane surface which remains constant even when fragmented material is continuously created. For this purpose, the flat aluminum surface was machined to have a corrugated finish. Any debris produced by the slipping action could become trapped within a neighboring crevice and therefore could not influence the subsequent motion of the steel cylinder. Results from this type of aluminum surface are presented in Fig. 7 to reveal the free-rolling actions and in Fig. 8 to compare theory with experiment for the rolling plus slipping motion. Both figures show data obtained using three cylinders whose moments of inertia are 2.45×10^{-5}, 2.22×10^{-5} and 1.01×10^{-5} kg m^2.

In Fig. 7 it is seen that the values of the moment of inertia for the cylinders account for the different accelerations for any fixed angle β less than the critical angle β_0. The precision of the data is sufficiently accurate to show that for any free-rolling motion the measured values of the acceleration are slightly less than that predicted by eqn. (3) of the theory.



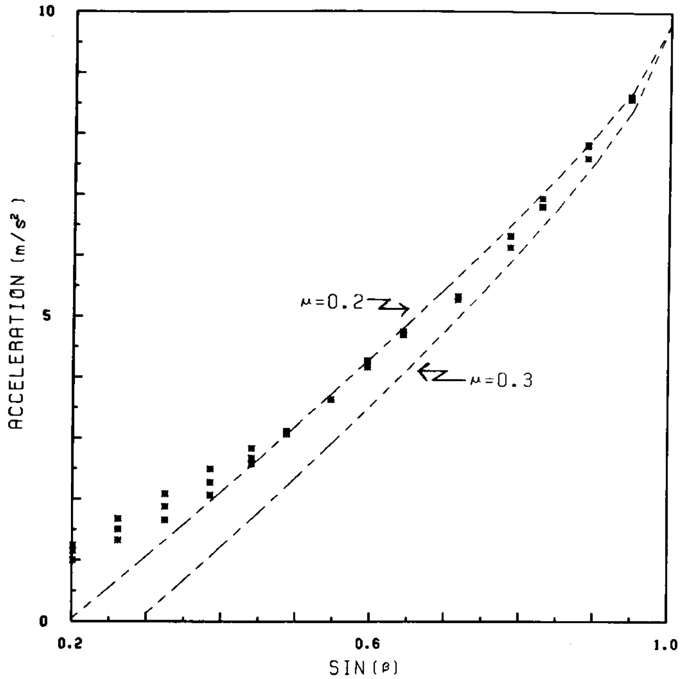


Fig. 7. A comparison is made between theory (——) and experiment (✱) for the case of free rolling on a corrugated surface. The different lines designate various values of the moment of inertia, and the measured accelerations have magnitudes slightly less than those predicted. These variations between the lines and the data for small angles are attributed to rolling friction.

Fig. 8. Three sets of values of acceleration are given for each angle of the inclined plane in accordance with the three moments of inertia of the cylinders. Once rolling and slipping take place on the corrugated surface, the acceleration is independent of the moment of inertia because each cylinder has the same magnitude of acceleration. The experimental data do not agree with the theoretical data (— — —) determined for a constant coefficient of friction.

Evidently, this variation between experiment and theory is associated with rolling friction [6].

Our most precise values for the acceleration of cylinders *versus* sin β for the simultaneous actions of rolling and slipping are shown in Fig. 8. At each angle examined, measurements of the acceleration were made and are plotted for the three aforementioned cylinders. The data indicate that once slipping occurs there is coincidence of the experimental points for the various cylinders and, accordingly, the slipping action must be independent of the moment of inertia. Also, the experimental values fit the theory only for $\mu < 0.2$ when sin $\beta < 0.5$ and $\mu > 0.2$ when sin $\beta > 0.7$.

The various evaluations of the critical angles and their corresponding coefficients of kinetic friction are listed in Table 3. For any one type of aluminum surface the values of μ_0 are quite consistent, but as the finish of the aluminum is altered μ_0 also varies such that the lowest value occurs for the corrugated surface.

TABLE 3

The critical angles and the corresponding coefficients of kinetic friction for the different types of aluminum surfaces and steel cylinders

	sin β_0	μ_0
Smooth surface		
Solid	0.53	0.21
Annular 1	0.49	0.20
Annular 3	0.44	0.21
Rough surface		
Solid	0.55	0.22
Annular 2	0.52	0.24
Annular 3	0.45	0.22
Corrugated surface		
Solid	0.50	0.19
Annular 2	0.43	0.19
Annular 4	0.37	0.19

4. Discussion

For spheres or cylinders, a plot of the linear acceleration of the moving object *versus* the sine of the angle of inclination of the path shows that there is a definite critical angle β_0 above which free rolling is converted into a rolling plus slipping behavior. One can readily describe both types of motion theoretically. When the rolling acceleration equals that due to rolling and slipping, the coefficient of friction can be evaluated in terms of the critical angle and the geometrical properties of the cylinder as given by eqn. (6).

From the measurements of the time intervals for cylinders progressing definite distances and by using eqn. (7) it is possible to calculate accurate values of acceleration since the data are quite reproducible and show good agreement with the theory for the rolling motion. However, there is considerable scatter of data points whenever slipping occurs. This error is attributed to the non-reproducible nature of the sliding surfaces since the aluminum plane is subject to wear.

The finish of the aluminum surface does influence the value of the coefficient of kinetic friction at the critical angle μ_0. As presented in Table 3, the smooth surface has a coefficient of friction near 0.21. The value is increased for the rough surface but is reduced to 0.19 for a corrugated surface. One may speculate that the value of μ_0 is partially dependent on the amount of debris between the interacting surfaces. Thus a corrugated plane offers less opposition to motion than a flat surface since wear fragments can be removed from the interacting surfaces almost as quickly as the chips are formed. However, it is quite likely that one simple assumption cannot completely justify all the variations in μ_0 since the amount of wear which takes place can also be an important factor.

The cylinders which were used have moments of inertia about the central axis which vary by a factor of 2.5, but there is no detectable correlation between the moment of inertia and the coefficient of friction. One should recognize, however, that the maximum linear speed involved is less than 1.5 m s^{-1}, and it may be possible for some effects of the moment of inertia to become evident at greater speeds.

For the rough as well as the corrugated surfaces there is a pronounced variation in the coefficient of friction as the angle of the inclined plane increases above β_0. An evaluation of μ was made by using the experimental data and eqn. (5), and the results of these calculations are plotted as functions of $\sin\beta$ in Fig. 9. Basically, for the rough surface μ varies from 0.22 to 0.34 while that for the corrugated surface changes from 0.19 to 0.24, and the percentage change is appreciable in both cases. There may also be some small variation in μ for the smooth surface, but the effect is not as definite as in the other two cases.

At all times, the motion of the steel cylinder over the aluminum surface is consistent with the existence of rolling friction because for all values of $\beta < \beta_0$ the measured acceleration for free-rolling motion is always slightly less than the theoretical prediction. Rolling friction is, in itself, a complex phenomenon [7] which includes elastic hysteresis, plastic deformation and microslip. When sliding is added to the rolling motion for an incline that exceeds the critical angle β_0, a macroscopic slipping action is included. This macroscopic slipping during dry friction may include shearing of asperities as well as ploughing, with a resulting increase in the force of friction. Since the magnitudes of the various types of interaction could easily change when the angle of inclination is increased to values greater than β_0, it is perfectly reasonable for the coefficient of friction to be a direct function of the angle of the inclined plane. By using smooth, rough

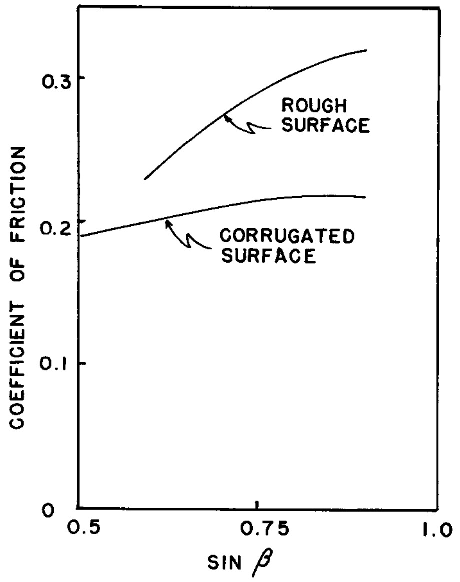


Fig. 9. To obtain agreement between theory and experiment for the rolling and slipping actions of cylinders, the magnitude of the coefficient of friction is regarded as a variable which depends on the angle of the inclined plane. The curved lines give the required functional variation in the coefficient of friction with the angle.

or corrugated inclined planes, it was shown that a variation in the magnitude of the frictional forces can account for the observed behavior shown in Fig. 9.

These results show that the coefficient of kinetic friction can be altered by the type of surface as well as the angle of inclination. The amount of wear for tribological surfaces is certainly governed by physical conditions, but before one can make any quantitative evaluation a precise determination of the nature of the interacting surfaces should be established in terms of their surface topology [8]. Besides evaluating the importance of the physical properties of surfaces, it is also practical to apply our simple experimental technique for the evaluation of the coefficient of friction of alloys, insulators and viscoelastic materials for both dry and lubricated surfaces.

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