Widely Separated MIMO Radar with Adaptive Waveform for Target Classification

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***Abstract*—In prior work, we have shown the advantage of adaptive waveforms for monostatic radar target recognition performance. In this paper, we extend our approach to the widely separated multi-input multi-output (MIMO) radar scenario. MIMO radar exploits a diversity of radar waveforms and target scattering to improve radar performance. We present an iterative information-based waveform design method that results in waveforms having narrow, disjoint bands so as not to interfere with each other. Applying a constant modulus constraint causes spectral spreading, but we study the impact of this spreading and find that the performance loss is minimal.**

I. INTRODUCTION

Multiple-input multiple-output (MIMO) radar [1] is a radar system that has multiple radar transmitters and receivers. Since there are multiple transmitters, the radar system can transmit multiple different waveforms, thus exploiting waveform diversity. However, full effectiveness of MIMO radar is achieved when the waveforms are carefully designed such that they can be separated at the receivers. In [2], MIMO radar with collocated antennas was considered. MIMO radar with collocated antennas gives several advantages compared to a phased-array, such as capability to handle more targets and flexibility of beampattern design. In [1], MIMO radar with widely separated antennas was reviewed. In this scenario, MIMO radar exploits the spatial diversity of radar cross section, resulting in improved radar performance in some cases.

Cognitive radar [3] is a new radar framework that uses adaptation not only in the receiver, but also in the transmitter. The adaptation in the transmitter is implemented by a feedback-loop from the receiver to the transmitter. Cognitive radar gives improved performance in various applications. For target recognition [4], cognitive radar can adapt its waveform via changing target probabilities, which results in lower energy use and quicker decisions, on average, than a radar transmitting fixed, conventional waveforms. For the target tracking scenario in [5], a cognitive radar was designed to pick a radar waveform from a prescribed library of waveforms, which reduced tracking error compared to traditional radar.

We extend our previous monostatic cognitive radar target recognition scenario to the MIMO radar framework. Two monostatic radars that observe the target from different aspects cooperate together to classify a target. Adaptive waveform design information is shared between the two radars such that the waveforms do not, at least initially, have overlapping frequency bands. However, once a constant-modulus constraint is applied to the time-domain waveform, spectral leakage caused by this constraint causes minimal spectral overlap and interference. We use template-based classification to evaluate the target recognition performance of this scenario.

The organization of the paper is as follows. In Section II, we describe the problem statement and signal model. In Section III, we summarize the radar waveform design technique. In Section IV, we show simulation results, and in Section V, we make our conclusions.

II. PROBLEM STATEMENT AND SIGNAL MODEL

Two monostatic radar systems, denoted as radar *A* and radar *B*, are applied to a target classification scenario. It is assumed that the two radars are located in different places and observe the target from significantly different angles, but are connected by a network to share waveform spectrum information as well as the results from processing the received signals. It is also assumed that both radars have target template libraries for *M* potential target types stored in the system. The radars search an area to detect a target, and now the goal is to choose which of the *M* target types is present. The targets have an infinite number of unique signatures that vary with azimuth and elevation angle. To reduce complexity in the analysis, for this paper we only consider azimuth angle variation; thus, the elevation angle is fixed.

To classify the target, radars *A* and *B* transmit the waveforms $s_A(t)$ and $s_B(t)$, respectively. The received signal $y_A(t)$ at radar *A* is

$$y_A(t) = g_A(t) * s_A(t) + h(t) * s_B(t) + n(t) \tag{1}$$

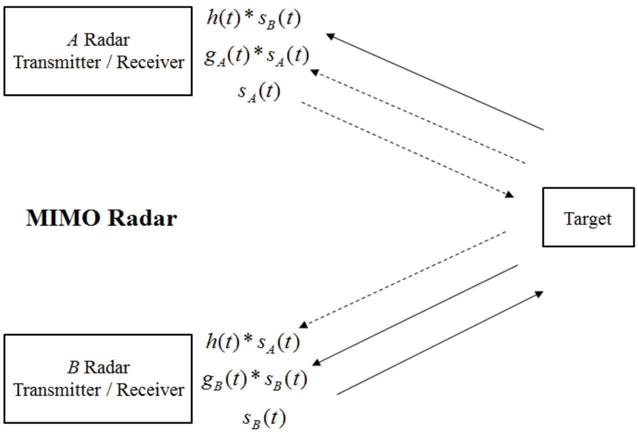

Fig. 1. MIMO radar system block diagram.

where $g_A(t)$ is the monostatic radar target signature from the perspective of radar A, $h(t)$ is the bistatic radar target signature, the symbol $*$ denotes convolution, and $n(t)$ is additive white Gaussian noise (AWGN) with power σ_n^2. Similarly, the received signal at radar B is

$$y_B(t) = g_B(t) * s_B(t) + h(t) * s_A(t) + n(t)\,. \tag{2}$$

A diagram showing the signal paths is shown in Figure 1.

In order to implement in a computer simulation, we need a discrete-time signal model. We sample the transmit waveforms, target signatures, and received noise to obtain a matrix-vector notation. Convolution between waveform and target can be represented as matrix multiplication via a properly formatted signal matrix [6], which yields received signal vectors at radars A and B according to

$$\mathbf{y}_A = \mathbf{S}_A \mathbf{g}_A + \mathbf{S}_B \mathbf{h} + \mathbf{n} \tag{3}$$

and

$$\mathbf{y}_B = \mathbf{S}_B \mathbf{g}_B + \mathbf{S}_A \mathbf{h} + \mathbf{n}\,. \tag{4}$$

To classify a target, the received signals are compared to the signals that are expected for a given waveform, target type, and orientation. To overcome complexity issues in the classifier, the target azimuth angle is divided into N_g uniform angular sectors. Within each sector, several target signatures are averaged to compute a mean-template signature for that angular sector. Since we have M possible target types, there are MN_g mean target templates that must be considered. These templates are defined as $\bar{\mathbf{g}}_i(t), i = 1,\ldots,N_g,\ldots,MN_g$. We assume that approximate orientation information can be obtained from an existing target track, such that the candidate angular sectors can be limited to just a few possibilities. We model the bistatic target signature as an impulse with zero-mean, complex-Gaussian reflectivity coefficient. This model is intended to represent poor range resolution for large bistatic angles, and therefore the bistatic path signal will introduce cross-platform interference of little or no value to the classification task.

Each angular sector for each target type is treated as a separate hypothesis. Therefore, conditioned on a particular target and orientation, the monostatic target template is deterministic, and the pdf of received signal $\mathbf{y}_A$ conditioned on a particular target-class/angle hypothesis is

$$p(\mathbf{y}_A \mid H_i) = \frac{\exp(-(\mathbf{y}_A - \boldsymbol{\mu}_{y,i}^A)^H (\mathbf{K}_{y,i}^A)^{-1} (\mathbf{y}_A - \boldsymbol{\mu}_{y,i}^A))}{(\pi)^N \,|\mathbf{K}_{y,i}^A|} \tag{5}$$

where $(\cdot)^H$ is the conjugate transpose operator. The, mean and covariance of $\mathbf{y}_A$ under the i^{th} target/angle hypothesis are $\boldsymbol{\mu}_{y,i}^A = \mathbf{S}_A \bar{\mathbf{g}}_i$ and $\mathbf{K}_{y,i}^A = \sigma_n^2 \mathbf{I} + \mathbf{S}_B E(\mathbf{h}\mathbf{h}^H)(\mathbf{S}_B)^H$, respectively. The pdf of $\mathbf{y}_B$ can be defined in a similar manner, but it is important to note that the bistatic target response $\mathbf{h}$ is identical for both bistatic path directions.

The bistatic path interference in the two received signal can be correlated due to the same realization of $\mathbf{h}$. However, if the transmit waveforms are in disjoint bands, this interference becomes uncorrelated. Thus, we assume independence in the two received signals, such that the combined pdf of the received signals is

$$p(\mathbf{y} \mid H_i) = p(\mathbf{y}_A \mid H_i)\, p(\mathbf{y}_B \mid H_i)\,. \tag{6}$$

Since the mean and covariance of the received signals depend on the transmitted waveforms, they must be updated as the waveforms adapt.

III. Radar Waveform Design

In [7], Bell derived the following result. If $g(t)$ is a Gaussian ensemble of target impulse responses, and the waveform is constrained in energy, duration, and bandwidth, then the waveform that maximizes the mutual information between the ensemble of target impulse responses and the received signal has the frequency spectrum given by [7]

$$|S(f)|^2 = \begin{cases} \max\left[0, A - \dfrac{\sigma_n^2 T_y}{2\sigma_G^2(f)}\right] & |f| \le \dfrac{1}{2T_s} \\ 0 & |f| > \dfrac{1}{2T_s} \end{cases} \tag{7}$$

where the ensemble's spectral variance function is represented by $\sigma_G^2(f) = E\{|G(f) - E\{G(f)\}|^2\}$, $G(f)$ is the Fourier transform of $g(t)$, and T_s is the discrete-time sampling interval. If the function $\sigma_n^2 T_y / 2\sigma_G^2(f)$ is defined as $r(f)$, the constant A controls the energy in the waveform according to

$$E = \int_{-1/2T_S}^{1/2T_S} \max\left[0, A - \frac{\sigma_n^2 T_y}{2\sigma_G^2(f)}\right] df \ . \qquad (8)$$

In our application, the ensemble of target templates is not Gaussian, but we have obtained good results by approximating the spectral variance function of our finite hypothesis ensemble according to [4]

$$\sigma_G^2(f) = \sum_{i=1}^{MN_g} P_i \,|\, \overline{G}_i(f) \,|^2 - |\sum_{i=1}^{MN_g} P_i \overline{G}_i(f) \,|^2 \qquad (9)$$

where $\overline{G}_i(f)$ is the Fourier transform of the i^{th} mean template and P_i is the probability that the i^{th} target/angle hypothesis is true. We then substitute this variance for the Gaussian ensemble's spectral variance in (7). The waterfilling [7] procedure is performed to find the solution to (7). We see in (7) that the waveform spectrum will be related to noise power and the (monostatic) spectral variance function. If spectral variance is high in a certain spectral band (indicating strong variability across the hypotheses), *r(f)* will be small in that band and the waterfilling procedure will allocate more energy.

In a MIMO radar scenario, we do not want the two radar waveform spectra interfering with each other. Thus, the radar waveforms should be spectrally non-overlapping. Fortunately, the information-based waveform design method tends to focus energy of a single waveform into only a few narrow spectral bands. Thus, our approach is to try a two-step waveform design procedure. First, we design the waveform for radar *A* without considering radar *B*'s waveform. The waveform for radar *B* is then designed, but with knowledge of the interference that will be observed due to radar *A*. In other words, radar *B* factors in the interference produced by radar *A* when optimizing its waveform. Therefore, radar *B* is less likely to put energy into the same bands as *A* radar is using.

Since the radar waveforms are designed in the frequency domain (see (7)), they do not have the constant modulus property [8] in the time domain. The goal of the waveform optimization is to make efficient use of transmit energy; hence, it is important to constrain the waveforms to be constant modulus. The paper [8] presented an iterative magnitude and amplitude projection method to obtain a constant-modulus waveform with approximately desired Fourier transform. We apply this procedure, which provides constant-modulus amplitude but degrades the optimum waveform spectrum.

Finally, since the waveform design depends on the ratio of noise power to spectral variance at every frequency, the radar range equation [9] should be used to compute received power. This received power is used to modify the spectral variance of the target ensemble according to target range [10].

IV. Results

We apply the technique above to a computer simulation of target classification. We assume that the radars must decide between two possible targets: an F-16 and an A-10. The elevation angle is -20˚, and target templates are generated every 0.1˚ over a 90˚ interval starting with a head-on configuration. The 90˚ interval is divided into 1˚ sectors, and the target signatures within each sector are averaged to calculate the mean templates. We assume that target track information is tight enough to limit target orientation to within two angular sectors. Since we have two target types and each type has two possible angular sectors, we have four possible templates for each radar, and four hypotheses from which to choose. The bistatic angle between the two radars and the target is 45˚, and the distance from each radar to the target is 20 km. The two radars transmit their waveforms and collect the return signals simultaneously. Upon collecting data, the probability of each target and each orientation are updated using the probability models described in Section II. The updated probabilities are then used to update the spectral variance in (9), which leads to newly optimized waveforms on the subsequent transmission. This procedure continues for a fixed number of transmissions, at which time a decision is made in favor of the target with the highest posterior probability. In the simulations below, we have fixed the number of transmissions at five.

We compare the performance of two different waveform strategies. The first is the optimization technique described above. The second waveform is a wideband waveform with energy spread evenly over the allowable bandwidth. Since the information-based waveforms are unlikely to overlap in frequency, they minimize interference between the radars. However, the wideband (impulse) waveforms have completely overlapping spectra, resulting in interference and performance loss.

We model an S-band radar with bandwidth of 250 MHz. Additional simulation parameters are an antenna gain of 30dB, effective noise temperature of 290K, noise power of about 1 pW, and average target RCS of 1 m^2. We performed 16,000 Monte Carlo simulations for error rates larger than $1\text{x}10^{-2}$ and 64,000 Monte Carlo simulations for error rates less than $1\text{x}10^{-2}$. For each Monte Carlo trial, we randomly select a new target orientation angle and generate new noise realizations. Figures 2 and 3 show waveforms obtained with the information-based technique before and after applying the constant-modulus constraint, respectively. In Figure 2, the ideal waveforms are completely orthogonal in frequency. In Figure 3, however, we see that the constant-modulus constraint has caused spreading of the waveform energy over frequency, such that the two waveforms now have some overlap. This overlap may cause some interference, especially

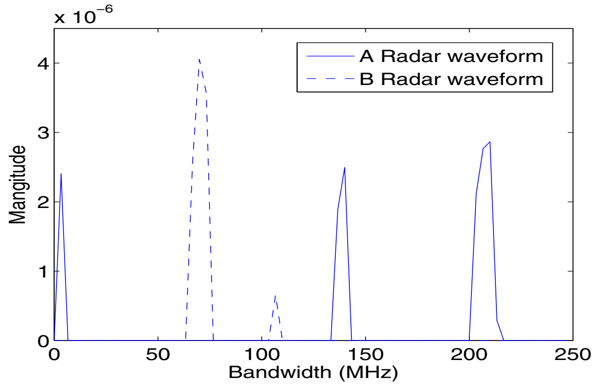

Fig. 2. Non-overlap waveform spectra before applying constant-modulus constraint.

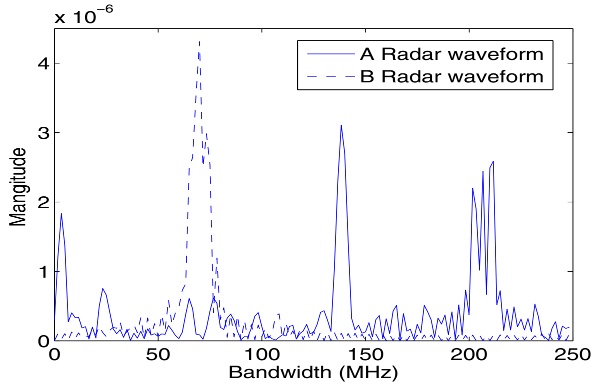

Fig. 3. Overlap waveform spectra after applying constant-modulus constraint.

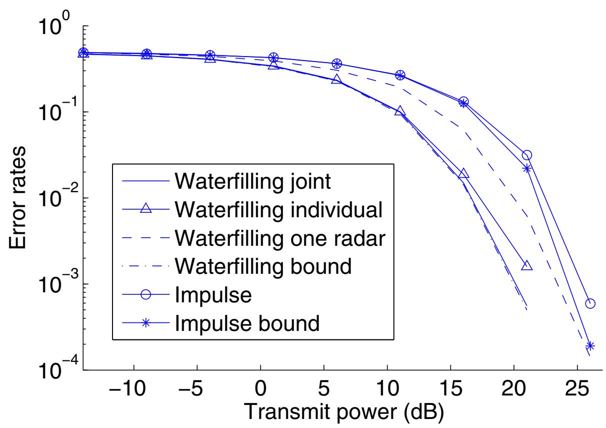

Fig. 4. Error rates versus transmit power for fixed number of iterations.

at higher transmit power. Figure 4 shows the resulting error rates versus transmit power. The *Waterfilling joint* denotes the case where the waveforms of both radars are designed jointly to minimize spectral overlap. *Waterfilling individual* denotes the case where the waveforms of each radar are designed individually, without knowledge of the other radar's waveform. The *bound* curves refer to a non-physical case where both radars can somehow ignore the interference due to the other transmitter. We also include single-radar performance for comparison. Surprisingly, the interference seems to have minimal impact. This behavior is likely due to the lower energy levels per spectral band for the wideband waveforms and due to the non-overlapping spectral design of the optimized waveforms. The individually optimized waveforms show some performance loss at high SNR due to occasional spectral overlap. Comparing the two primary waveform design strategies, the information-based waveforms show about 5 dB improvement compared to the performance of wideband waveforms.

V. CONCLUSION

We have implemented two widely separated MIMO radars with adaptive waveforms for a target recognition scenario. Information-based waveforms are designed to be spectrally non-overlapping, and therefore to avoid cross-platform interference. The analysis shows that any interference caused by spectral leakage due to constant-modulus constraints has minimal impact. The performance of an adaptive, information-based waveform design approach is compared to a standard wideband waveform, and it is seen that the adaptive, information-based waveform provides better performance. We conclude that information-based waveforms should work very well in this type of application due to their naturally sparse frequency spectra, which enables waveform orthogonality and reduces interference.

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