Improved waveform design for radar target classification

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The information-theoretic waveform for target classification based on spectral variance has been studied, and its advantages have been shown. The waveform design algorithm has a low computational load and can be applied to real-time applications. In looking for more sophisticated methods while keeping the advantages of an information-based approach, a new design algorithm by deriving a strict lower bound of the information measure is developed. The proposed design algorithm requires a small amount of calculation and shows better classification performance in terms of percentage of correct detection. The proposed method is compared with other methods, and the improved performance is shown in numerically generated graphs.

Introduction: A classification waveform algorithm was derived by defining mutual information based on energy spectral variance (MIESV) across the multiple target transfer functions in [1–3]. The energy spectral variance (ESV) is a statistical variance among the given target transfer functions, and the details are well documented in the literature. In this Letter, we develop a new waveform design algorithm by deriving a strict lower bound of MIESV (LBM) and optimising an enhanced waveform by maximising the lower bound. When the lower bound is a tight bound, the performance should be improved. That is, the maximisation of the lower bound means sufficiently the maximisation of MIESV. We verify the performance improvement of the LBM waveform method compared with the MIESV waveform method by computer simulation results.

System model and MIESV waveform algorithm: We adopt the similar stochastic extended target model as in [2, 3] as follows. The received signal is

$$y(t) = w(t) * h_i(t) + n(t) \tag{1}$$

where the target hypothesis index is $i = 1, 2, \ldots, \mathcal{H}$, $w(t)$ is a finite-energy waveform with duration T_w, $*$ denotes the convolution, $n(t)$ is the zero-mean receiver noise process with power spectral density (PSD) $P_n(f)$, and the random target $h_i(t)$ is a wide-sense stationary process with PSD $S_h(f)$. For a finite-duration stochastic target model, we consider a finite-target model $g_i(t) = a(t)h_i(t)$ where $a(t)$ is a rectangular window function of duration T_g [3]. We apply the Fourier transform to the target model to obtain a frequency-domain model as

$$\boldsymbol{y}(f) = \boldsymbol{w}(f)\boldsymbol{g}_i(f) + \boldsymbol{n}(f) \tag{2}$$

The target impulse response $\boldsymbol{g}_i(f)$ can be a deterministic or random target. For the random target model, $\boldsymbol{g}_i(f)$ is a finite-energy process with zero mean [3].

Now, let us review the MIESV waveform algorithm based on (2). MIESV was derived for two different target models [1–3]. The first is a deterministic target model and the second is a random target model. For the deterministic case, we have

$$\mathrm{MIESV_D} = T_y \int_B \ln\left[1 + \frac{|\boldsymbol{w}(f)|^2 S_\mathrm{D}(f)}{T_y P_n(f)}\right] \mathrm{d}f \tag{3}$$

where the ESV for a deterministic target model is $S_\mathrm{D}(f) = \sum_{i=1}^{\mathcal{H}} \Pr(H_i) |\boldsymbol{g}_i(f)|^2 - |\sum_{i=1}^{\mathcal{H}} \Pr(H_i)\boldsymbol{g}_i(f)|^2$, and $\boldsymbol{g}_i(f)$ is the frequency response of the ith target. $\Pr(H_i)$ is the probability that the ith target is present. For the random case, we have

$$\mathrm{MIESV_R} = T_y \int_B \ln\left[1 + \frac{|\boldsymbol{w}(f)|^2 S_\mathrm{R}(f)}{T_y P_n(f)}\right] \mathrm{d}f \tag{4}$$

where the ESV is $S_\mathrm{R}(f) = \sum_{i=1}^{\mathcal{H}} \Pr(H_i)\sigma_{g_i}^2(f) - |\sum_{i=1}^{\mathcal{H}} \Pr(H_i)\sqrt{\sigma_{g_i}^2(f)}|^2$, and $\sigma_{g_i}^2(f)$ is the spectral energy function of the ith target. The mutual information is then maximised with respect to the waveform under a waveform energy constraint $\int_B |w(f)|^2\, \mathrm{d}f \le E_\mathrm{max}$. In the next section, we improve on (3) and (4) by deriving their lower bounds.

Lower bound of MIESV: By inspecting the structure of MIESV, we can improve the information measure since the measure includes the sum of spectral energies and the mutual information is a concave function. Thus, Jensen's inequality provides a strict LBM. By this inequality operation, we obtain a lower bound and design a waveform by maximising the bound. Since the ESV is an approximation, the lower bound approach is a stricter measure. When the bound is tight enough, we can obtain a better waveform. Thus, we will perform and show computer simulations to see whether the waveform method based on LBM yields better performance or not. Next, we focus on the LBM derivation for the random target model sets.

Before we derive LBM, we define a common function $\mu_\mathrm{R}(f)$ from the given $\mathcal{H}$-number of target impulse responses by $\mu_\mathrm{R}(f) = |\sum_{i=1}^{\mathcal{H}} \Pr(H_i) \sqrt{\sigma_{g_i}^2(f)}|^2$. The ESV is now expressed as $S_\mathrm{R}(f) = \sum_{i=1}^{\mathcal{H}} \Pr(H_i)\sigma_{g_i}^2(f) - \mu_\mathrm{R}(f)$. Now, we derive LBM as

$$\begin{aligned}
\mathrm{MIESV_R} &= T_y \int_B \ln\left[1 + \frac{|\boldsymbol{w}(f)|^2 S_\mathrm{R}(f)}{T_y P_n(f)}\right] \mathrm{d}f \\
&= T_y \int_B \ln\left[1 + \frac{|\boldsymbol{w}(f)|^2 \{\sum_{i=1}^{\mathcal{H}} \Pr(H_i)\sigma_{g_i}^2(f) - \mu_\mathrm{R}(f)\}}{T_y P_n(f)}\right] \mathrm{d}f \\
&= T_y \int_B \ln\left[1 + \frac{|\boldsymbol{w}(f)|^2 \{\sum_{i=1}^{\mathcal{H}} \Pr(H_i)\sigma_{g_i}^2(f) - \sum_{i=1}^{\mathcal{H}} \Pr(H_i)\mu_\mathrm{R}(f)\}}{T_y P_n(f)}\right] \mathrm{d}f \\
&= T_y \int_B \ln\left[\sum_{i=1}^{\mathcal{H}} \Pr(H_i)\cdot 1 + \sum_{i=1}^{\mathcal{H}} \Pr(H_i)\left\{\frac{|\boldsymbol{w}(f)|^2 \{\sigma_{g_i}^2(f) - \mu_\mathrm{R}(f)\}}{T_y P_n(f)}\right\}\right] \mathrm{d}f
\end{aligned}$$

since $\sum_{i=1}^{\mathcal{H}} \Pr(H_i) = 1$

$$= T_y \int_B \ln\left[\sum_{i=1}^{\mathcal{H}} \Pr(H_i)\left\{1 + \frac{|\boldsymbol{w}(f)|^2 \{\sigma_{g_i}^2(f) - \mu_\mathrm{R}(f)\}}{T_y P_n(f)}\right\}\right] \mathrm{d}f \tag{5}$$

where $\mu_\mathrm{R}(f)$ does not depend on the index i. Applying Jensen's inequality yields

$$\mathrm{MIESV_R} \ge \sum_{i=1}^{\mathcal{H}} \Pr(H_i)\left\{T_y \int_B \ln\left[1 + \frac{|\boldsymbol{w}(f)|^2 \{\sigma_{g_i}^2(f) - \mu_\mathrm{R}(f)\}}{T_y P_n(f)}\right] \mathrm{d}f\right\}$$

since the logarithm function is a concave function. Therefore, the final optimisation problem is

$$\begin{aligned}
&\max \sum_{i=1}^{\mathcal{H}} \Pr(H_i)\left\{T_y \int_B \ln\left[1 + \frac{|\boldsymbol{w}(f)|^2 \{\hat{\sigma}_{g_i}^2(f)\}}{T_y P_n(f)}\right] \mathrm{d}f\right\} \\
&\text{s.t.} \int_B |\boldsymbol{w}(f)|^2\, \mathrm{d}f \le E_\mathrm{max}
\end{aligned} \tag{6}$$

where $\hat{\sigma}_{g_i}^2(f) = \sigma_{g_i}^2(f) - \mu_\mathrm{R}(f)$. The objective function of the optimisation procedure is the LBM, and its structure is composed of the sum of multiple mutual information measures. Each mutual information is defined based on the ith filtered spectral energy function $\hat{\sigma}_{g_i}^2(f)$ as shown in (5). The ith filtered spectral energy function is the remnant spectral energy in which the common spectral energy $\mu_\mathrm{R}(f)$ has been removed. Classification is a process of emphasising the difference between the target signatures as opposed to their shared spectral information. Thus, it is reasonable to remove the common spectral information for target classification waveform design.

For computer simulations, we discretise the LBM optimisation problem of (5) into $\mathcal{L}$ frequency bins as

$$\begin{aligned}
&\max T_y \sum_{i=1}^{\mathcal{H}} \Pr(H_i) \sum_{l=1}^{\mathcal{L}} \ln\left[1 + \frac{|\boldsymbol{w}(l)|^2 \{\sigma_{g_i}^2(l) - \mu_\mathrm{R}(l)\}}{T_y P_n(l)}\right] \Delta f \\
&\text{s.t.} \sum_{l=1}^{\mathcal{L}} |\boldsymbol{w}(l)|^2 \le E_w
\end{aligned} \tag{7}$$

where Δf is the width of a frequency bin, $\mathcal{L}\Delta f$ is the transmit bandwidth, l is a discrete frequency index, and $\mu_\mathrm{R}(l) = |\sum_{i=1}^{\mathcal{H}} \Pr(H_i)\sqrt{\sigma_{g_i}^2(l)}|^2$.

Lagrangian optimisation: For the optimal waveform of LBM, we take Lagrangian multiplier method and obtain

$$L(\lambda, \Omega(l)) = T_y \sum_{i=1}^{\mathcal{H}} \Pr(H_i) \sum_{l=1}^{\mathcal{L}} \ln\left[1 + \frac{\Omega(l) S_i(l)}{T_y P_n(l)}\right] \Delta f + \lambda\left(\sum_{l=1}^{\mathcal{L}} \Omega(l) - E_w\right)$$

where $S_i(l) = \sigma^2_{\boldsymbol{g}_i}(l) - \mu_R(l)$, and $|\boldsymbol{w}(l)|^2 = \Omega(l)$. Here, let us apply the first derivative to $L(\lambda, \Omega(l))$ with respect to $\Omega(l)$ to obtain

$$\frac{\partial L(\lambda, \Omega(l))}{\partial \Omega(l)} = T_y \sum_{i=1}^{\mathcal{H}} \Pr(H_i) \frac{S_i(l)/T_y P_n(l)}{1 + \Omega(l)(S_i(l)/T_y P_n(l))} \Delta f + \lambda = T_y \sum_{i=1}^{\mathcal{H}} \Pr(H_i) \frac{A_i(l)}{1 + \Omega(l) A_i(l)} \Delta f + \lambda$$

where $A_i(l) = S_i(l)/T_y P_n(l)$. For the maximum information value of the objective function, we set $\partial L(\lambda, \Omega(l))/\partial \Omega(l) = 0$ to obtain

$$\sum_{i=1}^{\mathcal{H}} \Pr(H_i) \frac{A_i(l)}{1 + \Omega(l) A_i(l)} = \frac{\lambda}{T_y \cdot \Delta f}$$

Therefore, the optimal $\{\Omega(l) | l = 1, 2, \ldots, \mathcal{L}\}$ is numerically calculated by

$$\sum_{l=1}^{\mathcal{L}} \Omega(l) = E_w \quad \text{and} \quad \sum_{i=1}^{\mathcal{H}} \Pr(H_i) \frac{A_i(l)}{1 + \Omega(l) A_i(l)} = C \qquad (8)$$

where $C = \lambda/(T_y \cdot \Delta f)$, and $\Omega(l)$ is inversely proportional to C. The final optimal waveform is obtained by the relationship $|\boldsymbol{w}(l)|^2 = \Omega(l)$. This optimisation procedure is obtained via an iterative water-filling method [4].

Similarly, the waveform optimisation for the deterministic target model is

$$\max T_y \sum_{i=1}^{\mathcal{H}} \Pr(H_i) \sum_{l=1}^{\mathcal{L}} \ln\left[1 + \frac{|\boldsymbol{w}(l)|^2 \{|\boldsymbol{g}_i(l)|^2 - \mu_D(l)\}}{T_y P_n(l)}\right] \Delta f$$
$$\text{s.t. to } \sum_{l=1}^{\mathcal{L}} |\boldsymbol{w}(l)|^2 \le E_w \qquad (9)$$

where $\mu_D(l) = |\sum_{i=1}^{\mathcal{H}} \Pr(H_i) \boldsymbol{g}_i(l)|^2$. We can obtain the optimal waveform by a similar method.

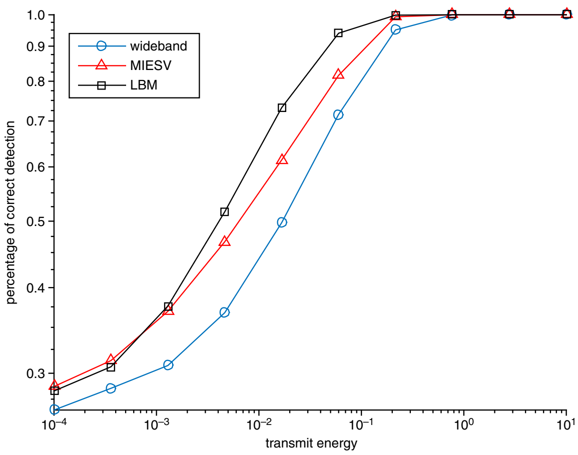

Fig. 1 *Performance comparison of various waveform design methods in case of 40 tap waveform with deterministic target set*

Simulation results: We show two simulation results that indicate the benefit of using LBM as a waveform design measure for a deterministic target model and a random target model. The system parameters for the simulation results are as follows. The waveform dimension $\mathcal{L}$ is 40, the measurement noise power σ^2 is normalised to 1, and the waveform energy allocation varies from 10^{-4} to 10^1 energy units. The number of target classes $\mathcal{H}$ is 4. The percentage of correct detection is calculated over 20,000 Monte Carlo trials. For Fig. 1, we averaged the results of ten different deterministic target sets. For the random target results in Fig. 2, we generated unique target class energy spectra for each trial, and then generated a realisation of the target from the energy spectrum of the correct target class. The percentage of correct detection is calculated in determining the true target transfer function for three different waveforms and their performances in Figs. 1 and 2 are compared. For comparison, we plot the results of two additional waveform methods: wideband waveform and the MIESV waveform. The wideband waveform has a uniform energy distribution over the transmission band. The LBM waveform shows the best performance among wideband, MIESV, and LBM waveforms and demonstrates a wider performance gain gap in the random target simulation of Fig. 2 than in the deterministic target simulation of Fig. 1. Especially, the result of random target simulation shows better improvement in the lower bound approach at just $<10^{-1}$ transmit energy units.

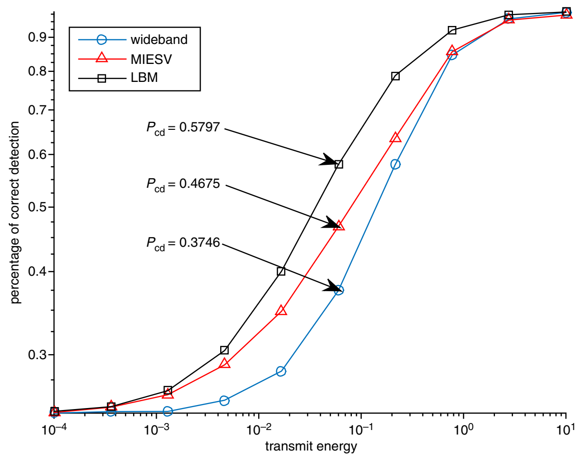

Fig. 2 *Performance comparison of various waveform design methods in case of 40 tap waveform with random target set*

Conclusion: An enhanced waveform design method has been derived by applying Jensen's inequality to MIESV. The proposed waveform design algorithm shows improved performance in computer simulations because the waveform is optimised by maximising the LBM measure, which is a strict LBM. The optimisation procedure is performed by an iterative water-filling method whose computational load is very light.

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One or more of the Figures in this Letter are available in colour online.

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References

1 Goodman, N.A., Venkata, P.R., and Neifeld, M.A.: 'Adaptive waveform design and sequential hypothesis testing for target recognition with active sensors', *IEEE J. Sel. Top. Signal Process.*, 2007, **1**, (1), pp. 105–113
2 Romero, R.A., and Goodman, N.A.: 'Waveform design in signal-dependent interference and application to target recognition with multiple transmissions', *IET Radar Sonar Navig.*, 2009, **3**, (4), pp. 328–340
3 Romero, R.A., Bae, J., and Goodman, N.A.: 'Theory and application of SNR and mutual information matched illumination waveforms', *IEEE Trans. Aerosp. Electron. Syst.*, 2011, **47**, (2), pp. 912–927
4 Yu, W., Rhee, W., Boyd, S., and Cioffi, J.M.: 'Iterative water-filling for Gaussian vector multiple-access channels', *IEEE Trans. Inf. Theory*, 2004, **50**, (1), pp. 145–152