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Classification waveform optimization for MIMO radar

Hyoung-soo Kim[1], **Nathan A. Goodman**[2], **Junhyeong Bae**[3], **and Chankil Lee**[4a)]

[1] *Department of Electronics Engineering, Korea Polytechnic University, 237, Sangidaehak-ro, Siheung-si, Gyeonggi-do 157073, Republic of Korea*

[2] *School of Electrical and Computer Engineering, University of Oklahoma, 110 W. Boyd St. Norman, Oklahoma 73019–1102, USA*

[3] *Agency for Defense Development, Daejeon, Republic of Korea*

[4] *Department of Electrical and Computer Engineering, Hanyang University, 55 Hanyangdaehak-ro, Sangnok-gu, Ansan, 15588, Republic of Korea*

a) *cklee@hanyang.ac.kr*

Abstract: The waveform algorithm for active target classification or identification based on spectral variance has been studied in various pieces of literature. The algorithm was extended for widely-separated MIMO Radar system to take advantage of spatial diversity gain by Bae et al. However, the MIMO waveform can be improved further by considering multiple objective functions from the multiple target paths. In this letter, we optimize the MIMO waveform for target identification system by maximizing the multiple objective functions. We show simulation results to compare the proposed algorithm to other MIMO waveform design methods.

Keywords: radar waveform, MIMO, target classification, mutual information, extended target

Classification: Sensing

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1 Introduction

Goodman et al. derived a classification waveform algorithm for enhanced target classification by defining mutual information based on energy spectral variance (MIESV) across the transfer functions of the various target hypotheses in [1, 2, 3]. Energy spectral variance (ESV) quantifies the statistical variance over a set of finite-duration target transfer functions. The details on the use of MIESV for classification task are described in [2, 3]. Bae et al. extended the MIESV algorithm for multiple-input, multiple-output (MIMO) radar by adding the spectral variance of the monostatic target impulse response and the spectral variances of the bistatic target impulse responses in [4] and showed the diversity gain in MIMO radar simulations.

In a MIMO radar system, multiple waveforms from widely separated radars are transmitted and reflected by a target. Then, the reflected waveforms are captured and combined by multiple radar receivers. Each single waveform is simultaneously captured by multiple radar receivers. For single-input, multiple-output (SIMO) waveform optimization, the single waveform affects the observations of the multiple radar receivers as shown in Fig. 1, and the waveform optimization can be performed by maximizing multiple objective functions based on the multiple observations. In addition, MIESV has the form of mutual information, and a MIMO waveform can be designed from the sum of the multiple mutual information measures. This information measure for each MIMO path is a function of the waveform parameters. Thus, we propose a MIMO classification waveform method based on multi-objective optimization (MO).

2 Monostatic radar model

We consider a stochastic extended target model for a particular target j, according to [2, 3]

$$y(t) = w(t) * h_j(t) + n(t) \tag{1}$$

where $*$ denotes convolution, the target hypothesis index $j = 1, 2, \ldots, \mathcal{H}$, $y(t)$ is the received observation with duration T_y, $w(t)$ is a finite-energy waveform with duration T_w, $n(t)$ is zero-mean receiver noise process with power spectral density (PSD) $P_n(f)$, and the random target $h_j(t)$ is a wide-sense stationary process with PSD $S_h(f)$. For a finite-duration stochastic target model, we adopt a finite target model $g_j(t) = a(t)h_j(t)$ where $a(t)$ is a rectangular window function of duration T_g [3]. The frequency-domain system model that results from the Fourier transform is

$$\mathbf{y}(f) = \mathbf{w}(f)\mathbf{g}_j(f) + \mathbf{n}(f)$$

where $\mathbf{g}_j(f)$ is a finite-energy process with zero-mean [3]. The classification waveform optimization based on MIESV is expressed by

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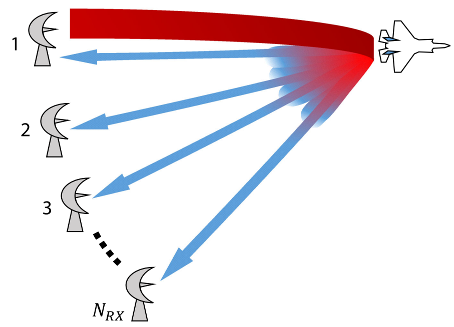

Fig. 1. SIMO radar waveform transmission and reception: The 1st radar transmits a waveform, and all radars receive a reflection

$$\max \text{MIESV}$$
$$\text{subject to} \int_B |\mathbf{w}(f)|^2 df \leq E_{\max} \tag{2}$$

where $\text{MIESV} = T_y \int_B \ln\left[1 + \frac{|\mathbf{w}(f)|^2 S_R(f)}{T_y P_n(f)}\right] df$, $E_{\max}$ is a given waveform energy constraint for $\mathbf{w}(f)$, the ESV for a random target model is $S_R(f) = \sum_{j=1}^{\mathcal{H}} Pr(H_j)\sigma^2_{\mathbf{g}_j}(f) - \left|\sum_{j=1}^{\mathcal{H}} Pr(H_j)\sqrt{\sigma^2_{\mathbf{g}_j}(f)}\right|^2$, $Pr(H_j)$ is the probability of the j^{th} target hypothesis, and $\sigma^2_{\mathbf{g}_j}(f)$ is the spectral energy of the j^{th} target. Now, let us define a MIMO radar signal model for use in waveform design.

3 MIMO radar model

For MIMO radar model, the observation of the k^{th} radar receiver due to the waveform from the l^{th} radar transmitter is defined as

$$\mathbf{y}_{kl}(f) = \mathbf{w}_l(f)\mathbf{g}_{jkl}(f) + \mathbf{n}_{kl}(f)$$

where the radar receiver index is $k = 1, 2, \ldots, N_{\text{Rx}}$, the radar transmitter index is $l = 1, 2, \ldots, N_{\text{Tx}}$, and $\mathbf{g}_{jkl}(f)$ is a finite-energy process with zero-mean [3]. Here, we have $N_{\text{Rx}} \times N_{\text{Tx}}$ number of observations. Each observation includes one of $\mathcal{H}$ target impulse responses. The MIMO waveform is composed of N_{Tx} SIMO waveforms since the multiple waveforms from the different transmit radars are separable in the radar receivers. First, let us consider the SIMO waveform for the 1^{st} radar. Fig. 1 shows that the first radar transmits a waveform, and its reflections are captured by the N_{Rx} radar receivers. From this result, an optimization method based on N_{Rx} objective functions is derived.

Each of the objective functions is individually defined by an MIESV metric. Thus, the SIMO waveform optimization from the l^{th} radar is described by multiple-objective optimization problem as

$$\max \sum_{k=1}^{N_{\text{Rx}}} C_k T_y \int_B \ln\left[1 + \frac{|\mathbf{w}_l(f)|^2 S_{kl}(f)}{T_y P_n(f)}\right] df$$
$$\text{subject to} \int_B |\mathbf{w}_l(f)|^2 df \leq E_{\max} \tag{3}$$

where $S_{kl}(f) = \sum_{j=1}^{\mathcal{H}} Pr(H_j)\sigma^2_{\mathbf{g}_{jkl}}(f) - \left|\sum_{j=1}^{\mathcal{H}} Pr(H_j)\sqrt{\sigma^2_{\mathbf{g}_{jkl}}(f)}\right|^2$, and C_k defines the weights of the combination. We can define the weight factors depending on the

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degree of importance of each $\text{MIESV}_{(k,l)}$ for a given l. However, in this letter, we set the weighting factors to unity for simplicity.

To enable computer simulation, we now define a discrete optimization problem from (3) as

$$\max \sum_{k=1}^{N_{\text{Rx}}} T_y \sum_{m=1}^{\mathcal{M}} \ln\left[1 + \frac{\Omega_l(m) S_{kl}(m)}{T_y P_n(m)}\right] \Delta f$$
$$\text{subject to } \sum_{m=1}^{\mathcal{M}} \Omega_l(m) \leq E_w \tag{4}$$

where m is the index of a discrete frequency bin after the transmit bandwidth has been divided into intervals, Δf is the width of a single frequency bin, $|\mathbf{w}_l(m)|^2 = \Omega_l(m)$, and E_w is a given waveform energy constraint for $\mathbf{w}_l(m)$. The scalar sum of the multiple objective functions is a concave function since each of the objective functions is $\ln|\cdot|$ which is concave. The concavity property of the scalar sum of $\ln|\cdot|$ was also discussed in [5]. Thus, we define a Lagrangian problem and derive the optimization procedure as

$$L(\lambda, \Omega_l(m)) = \sum_{k=1}^{N_{\text{Rx}}} T_y \sum_{m=1}^{\mathcal{M}} \ln\left[1 + \frac{\Omega_l(m) S_{kl}(m)}{T_y P_n(m)}\right] \Delta f + \lambda \left(\sum_{m=1}^{\mathcal{M}} \Omega_l(m) - E_w \right).$$

By applying the first derivative to $L(\lambda, \Omega_l(m))$ with respect to $\Omega_l(m)$, we obtain

$$\frac{\partial L(\lambda, \Omega_l(m))}{\partial \Omega_l(m)} = \sum_{k=1}^{N_{\text{Rx}}} T_y \frac{\frac{S_{kl}(m)}{T_y P_n(m)}}{1 + \Omega_l(m) \frac{S_{kl}(m)}{T_y P_n(m)}} \Delta f + \lambda = \sum_{k=1}^{N_{\text{Rx}}} T_y \frac{A_k(m)}{1 + \Omega_l(m) A_k(m)} \Delta f + \lambda$$

where $S_{kl}(f) = \sum_{j=1}^{\mathcal{H}} Pr(H_j) \sigma^2_{\mathbf{g}_{jkl}}(f) - \left| \sum_{j=1}^{\mathcal{H}} Pr(H_j) \sqrt{\sigma^2_{\mathbf{g}_{jkl}}(f)} \right|^2$ and $A_k(m) = \frac{S_{kl}(m)}{T_y P_n(m)}$. For the maximum value of the linear sum of multiple objective functions, let us apply $\frac{\partial L(\lambda, \Omega_l(m))}{\partial \Omega_l(m)} = 0$ to get

$$\frac{\partial L(\lambda, \Omega_l(m))}{\partial \Omega_l(m)} = 0 \Rightarrow \sum_{k=1}^{N_{\text{Rx}}} \frac{A_k(m)}{1 + \Omega_l(m) A_k(m)} = \frac{\lambda}{T_y \cdot \Delta f}.$$

The optimal $\Omega_l(m)$ can be numerically calculated according to the equations

$$\sum_{m=1}^{\mathcal{M}} \Omega_l(m) = E_w \text{ and } \sum_{k=1}^{N_{\text{Rx}}} \frac{A_k(m)}{1 + \Omega_l(m) A_k(m)} = \acute{\lambda} \tag{5}$$

where $\acute{\lambda} = \frac{\lambda}{T_y \cdot \Delta f}$, $\Omega_l(m)$, and $\acute{\lambda}$ are inversely proportional. This optimization procedure is an iterative water-filling algorithm [5]. Finally, we can obtain the energy spectrum of the l^{th} optimal SIMO waveform, $W_l = [\mathbf{w}_l(1), \mathbf{w}_l(2), \ldots, \mathbf{w}_l(\mathcal{M})]^T$ by $|\mathbf{w}_l(m)|^2 = \Omega_l(m)$. For the MIMO waveform matrix $\mathbf{W}$, we calculate multiple individual SIMO waveforms as $\mathbf{W} = [W_1, W_2, \ldots, W_{\text{Tx}}]$.

4 Simulation result

In this section, we present simulation results showing the performance of the multi-objective optimization algorithm (MO). For the computer simulations, the random

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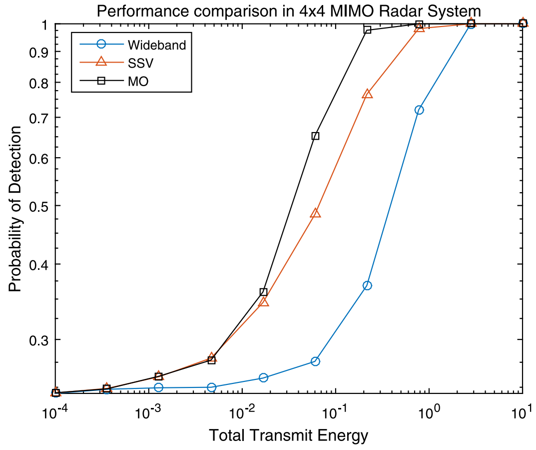

Fig. 2. Performance comparison in 4x4 MIMO radar system with colored target models

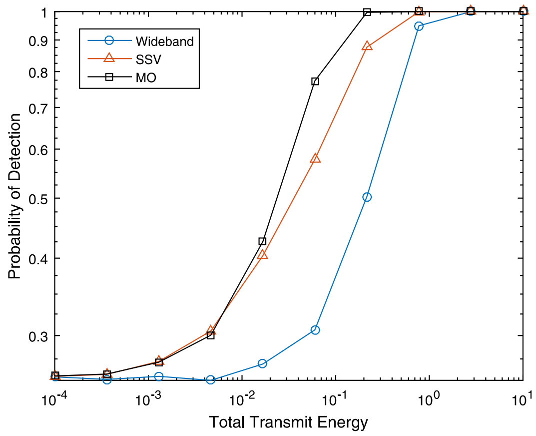

Fig. 3. Performance comparison in 4x4 MIMO radar system with non-colored target models

target signatures are generated in two different ways. The first model sets have ESV's generated from colored spectra. The second model sets have ESV's generated from flat (white) spectra. Also, we use the following system parameters. The number of target hypotheses $\mathcal{H}$ is 4, the discrete spectral waveform dimension $\mathcal{M}$ is 40, the measurement noise power σ^2 is normalized to 1, and the waveform energy allocation varies from 10^{-4} to 10^1 energy units. The probability of detection is calculated over 20,000 Monte Carlo trials. Based on the given system parameters, we evaluate the probability of detection in determining the true target transfer function for three different waveform algorithms and compare their performances in Figs. 2 and 3. The first waveform is a wideband waveform having a flat energy distribution across the transmission band, the second waveform is the waveform based on the sum of spectral variance (SSV) of [4], and the last waveform is the MO waveform that we propose in this letter. From the results, MO algorithm shows the best performance among the three waveform algorithms both in the cases of colored and non-colored target models. The results of Fig. 2 shows wider performance gain compared to that of Fig. 3 due to the more structured target spectra.

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5 Conclusion

We derived a MIMO waveform for classification via mutual information based on energy spectral variance. The waveform was optimized by the multi-objective optimization for the widely-separated MIMO radar model. The proposed method showed the best results in computer simulations, including simulations based on both flat and colored spectral models. As another advantage of the proposed algorithm, the optimization procedure is performed by an iterative water-filling algorithm whose computational load is very light. In this letter, we set the weighting factors C_k of (3) to unity for simplicity. However, it is also an interesting problem to find the best combination of the weighting factors since the factors are depending on the degree of importance of each MIESV metric and are related with the MIMO radar channel. This will be our future research topic.

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