Information-Theoretic Matched Waveform in Signal Dependent Interference

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***Abstract*—The design of transmit waveforms is critical to the performance of a radar system. Traditionally, pulsed and wideband chirp transmit waveforms are used. Our focus here is on the design of waveforms matched to extended targets. In the literature, optimum transmit waveform design in additive receiver noise has been investigated both from SNR maximization and information-theoretic approaches. In this paper, we extend the information-based approach to the signal-dependent interference problem. In particular, we investigate the use of information theory to generate the optimum waveform matched to a Gaussian distributed target ensemble with known spectral variance in the presence of signal-dependent clutter. Several waveform design examples are shown. We also apply this waveform technique to a cognitive radar application.**

***Index Terms*—Cognitive Radar, Matched Illumination, Information Theory**

I. Introduction

Cognitive radar has been proposed as a technological solution for performance optimization in resource-constrained and interference-limited environments [1], [2]. To that end, we consider a radar signal model in signal-dependent interference and receiver noise where the transmit waveform has an energy constraint. In [3], Bell proposed two energy-constrained waveform design problems and solutions in additive Gaussian channel noise from two different optimization approaches. The first approach considered maximization of signal-to-noise (SNR) ratio wherein a known target impulse response is used in the design of an optimal transmit-waveform/receiver-filter pair. The maximization of SNR leads to a transmit eigen-waveform solution. The second approach modeled the target impulse response as random and derived a waveform that maximizes mutual information between a Gaussian distributed target ensemble with a known spectral variance and the received signal in additive Gaussian noise. The information-theoretic approach leads to a waterfilling solution in the frequency domain. In [2] both approaches were demonstrated to improve performance of a closed-loop radar system performing target recognition. The two waveform design approaches were integrated with sequential hypothesis testing (SHT) to form a closed-loop active sensor. SHT was used to update the probabilities of multiple target hypotheses based on the received signal. Then, the two signal design approaches [3,5-6] were used to calculate the next transmit waveforms. When sufficient understanding of the channel was achieved, a target classification decision was made.

In [4-6], Pillai, et. al. considered the problem of matching a waveform to an known extended target in the presence of signal-dependent interference and receiver noise. It was proposed to jointly design a transmit waveform and a receiver impulse response by maximizing the output signal-to-interference-plus-noise ratio (SINR). Clearly the difficulty lies in the fact that the interference is signal-dependent due to the convolution of the clutter impulse response with the transmit waveform. While there was no closed-form solution derived, an iterative algorithm was proposed to jointly design the transmit waveform and receiver matched filter. Here, we apply the information-based approach to the signal-dependent interference problem. We derive the waveform that maximizes the mutual information between a Gaussian target ensemble and the received signal, wherein the received signal is composed of the convolution of the transmit waveform with the target and clutter impulse responses in addition to the receiver noise.

This paper is organized in the following manner. Section II describes the signal model for the noise-only case and summarizes the mutual-information-based waveform derived by [3]. Section III shows the derivation of the MI-based waveform for the signal-dependent interference. Section IV illustrates various waveform design examples. Section V shows the application of the signal-dependent-interference optimum waveform to a cognitive radar performing target recognition.

II. Signal Model and MI-Based Waveform Design

The block diagram in Fig. 1 represents the real-valued signal model being considered. Let $x(t)$ be a finite-energy waveform with duration T. Let $g(t)$ represent a Gaussian extended target ensemble with energy spectral variance $\sigma_G^2(f)$ as defined in [3], i.e., let T_g be the time duration where most of the target impulse's energy resides. It is necessary to have $T \geq T_g$ to capture the target impulse response's energy. The clutter $c(t)$ is a zero-mean Gaussian random process with spectral density $\sigma_C^2(f)$, and $n(t)$ is the zero-mean receiver noise process with one-sided PSD $P_n(f)$. Let $y(t)$ be the received signal given by

$$y(t) = x(t) * g(t) + x(t) * c(t) + n(t). \tag{1}$$

The signal-dependent interference is the clutter $c(t)$ convolved with the transmit waveform $x(t)$. We are interested in the mutual information (MI) between our measurement $y(t)$

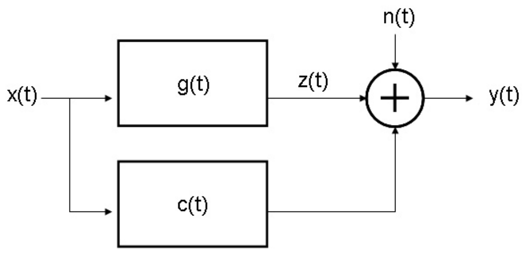

Fig. 1. Signal model of a Gaussian target ensemble in ground clutter

and target ensemble $g(t)$ given a transmit signal $x(t)$, i.e., $I(y(t); g(t)|x(t))$.

Bell derived the information-based waveform solution for the channel-noise-only case where $c(t) = 0$. The next few equations summarize the results of [3]. First, the mutual information $I(y(t); g(t)|x(t))$ is given by the equation

$$I(y(t); g(t)|x(t)) = T \int_W \ln \left[1 + \frac{2|X(f)|^2 \sigma_G^2(f)}{TP_n(f)} \right] df \quad (2)$$

where $|X(f)|^2$ is the energy spectrum of $x(t)$ and W is the bandwidth that contains most of its energy. Equation (2) is then maximized with respect to $|X(f)|^2$ under the constraint

$$E_x \geq \int_W |X(f)|^2 \, df. \quad (3)$$

Using the Lagrangian multiplier technique, maximization with respect to $|X(f)|^2$ leads to the following waterfilling waveform described by

$$|X(f)|^2 = max \left[0, A - \frac{TP_n(f)}{2\sigma_G^2(f)} \right] \quad (4)$$

where A is a constant determined by the energy constraint

$$E_x \geq \int_W max \left[0, A - \frac{TP_n(f)}{2\sigma_G^2(f)} \right] df. \quad (5)$$

Note that the function being "waterfilled" is the function $TP_n(f)/2\sigma_G^2(f)$ and A defines the water level. The constraint of (5) makes the MI-based waveform in (4) not necessarily finite time duration, but an approximately optimal finite-duration waveform is possible.

III. Information-Theoretic Waveform Design in Signal-Dependent Clutter

In this section, we re-consider the information-based approach in the presence of signal-dependent clutter. It can be shown that the mutual information $I(y(t); g(t)|x(t))$ is given by

$$I(y(t);g(t)|x(t)) = T \int_W \ln \left[1 + \frac{2|X(f)|^2 \sigma_G^2(f)}{TP_n(f) + 2|X(f)|^2 \sigma_C^2(f)} \right] df. \quad (6)$$

Note that the fraction in the right-hand side of the equation represents the SINR with the denominator containing the interference-plus-noise term. The interference-plus-noise term contains the transmit signal itself in the term $2|X(f)|^2 \sigma_C^2(f)$. To maximize the mutual information $I(y(t); g(t)|x(t))$, we maximize (6) with respect to $|X(f)|^2$ while conforming to the energy constraint in (3). Despite the additional interference term in (6), the function within the integral is easily confirmed to be concave by standard calculus techniques. Taking into account the energy constraint, the Lagrangian multiplier technique is invoked which leads to the waterfilling waveform described by

$$|X(f)|^2 = max \left[0, -R(f) + \sqrt{R^2(f) + S(f)(A - D(f))} \right] \quad (7)$$

where $D(f)$, $R(f)$ and $S(f)$ are defined by

$$D(f) = \frac{TP_n(f)}{2\sigma_G^2(f)}, \quad (8)$$

$$R(f) = \frac{TP_n(f)(2\sigma_C^2(f) + \sigma_G^2(f))}{4\sigma_C^2(f)(\sigma_C^2(f) + \sigma_G^2(f))}, \quad (9)$$

$$S(f) = \frac{TP_n(f)\sigma_G^2(f)}{2\sigma_C^2(f)(\sigma_C^2(f) + \sigma_G^2(f))}, \quad (10)$$

and A is a constant determined by the energy constraint

$$E_x \geq \int_W max \left[0, -R(f) + \sqrt{R^2(f) + S(f)(A - D(f))} \right] df \quad (11)$$

While the exact solution given by (7) will be used in the subsequent results section, it is somewhat hard to acquire an intuition of the waveform it describes. To gain further intuition, let $W(f)$ be defined by

$$W(f) = -R(f) + \sqrt{R^2(f) + S(f)(A - D(f))}. \quad (12)$$

Applying a first-order Taylor approximation to (12), the approximation yields

$$\tilde{W}(f) = B(f)(A - D(f)) \quad (13)$$

where $B(f)$ is

$$B(f) = \frac{\sigma_G^2(f)}{2\sigma_C^2(f) + \sigma_G^2(f)}. \quad (14)$$

Thus the waterfilling waveform is described by

$$|\tilde{X}(f)|^2 = max[0, B(f)(A - D(f))] \quad (15)$$

where A still controls the waveform's energy.

It clear that if $\sigma_C^2(f) = 0$, then (14) goes to unity and (15) becomes (4), which is the waveform solution to the channel-noise only case. For the case where clutter is non-zero, $B(f)$ is a clutter factor that affects both the waterfilling function $D(f)$ and the energy-controlling constant A . To see its effect, realize that $B(f)$ takes on non-negative real values between 0 and 1. When the clutter spectrum is zero, the clutter factor becomes one and has no effect. When the clutter factor is non-zero, each frequency component in $A - D(f)$ is weighted depending on the clutter spectrum. As clutter becomes strong in a certain frequencies, the clutter factor $B(f)$ goes to zero at those frequencies and no energy is utilized in those frequencies. Thus, the clutter factor $B(f)$ plays a major role in shaping of the first-order Taylor approximation of the optimum waveform. This will be explored more in the subsequent results section.

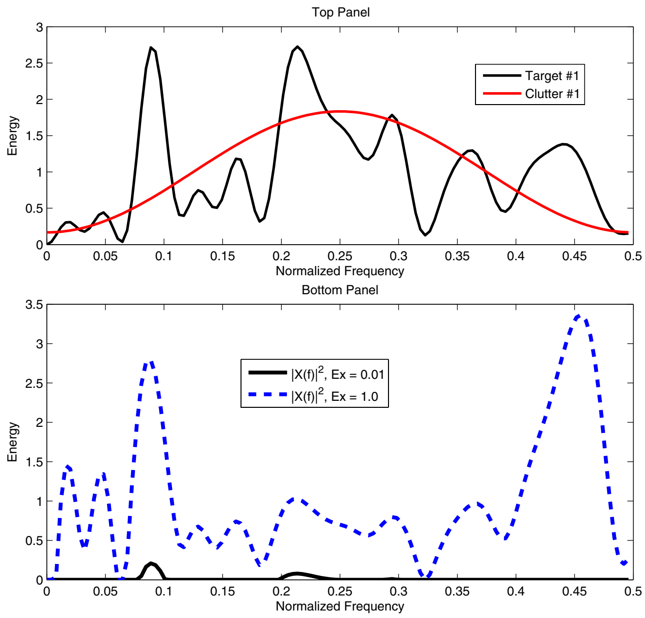

Fig. 2. Target #1, Clutter #1, and $|X(f)|^2$ with constraint Ex = 0.01, and Ex = 1.0

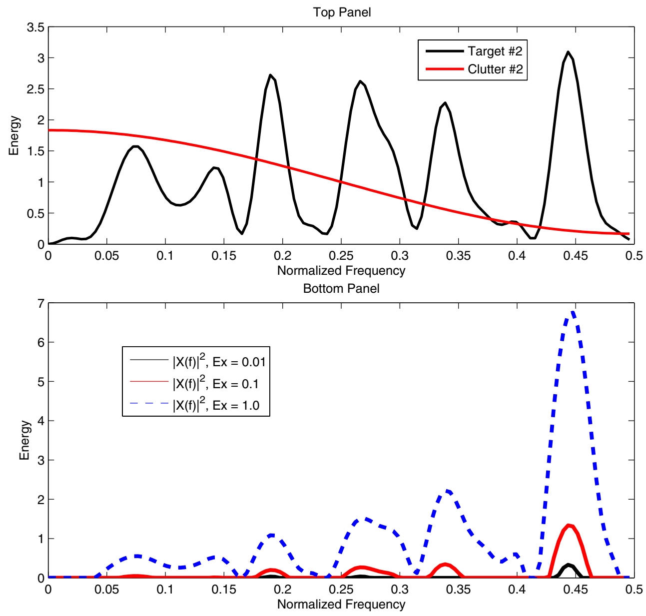

Fig. 3. Target #2, Clutter #2, and $|X(f)|^2$ as a function of increasing energy constraint Ex = 0.01, 0.1 and 1.0

IV. Waveform Design in Clutter: Examples

In this section, three design examples illustrating transmit waveform spectra obtained from the above waterfilling solution will be shown. The target, clutter, and noise processes are real valued. Since the spectra are symmetric, only the half-sided spectra will be shown. In practice, receiver front end noise is usually modeled as AWGN. Thus, we use a flat PSD to model the receiver noise in all subsequent experiments.

A. Target #1, Clutter #1, Constraints $E_x = 0.01,\ 1.0$

First, we consider an arbitrary target spectrum labeled Target #1 and a clutter spectrum labeled Clutter #1 shown in the top panel of Fig. 2. The first energy constraint is $E_x = 0.01$. The waterfilling action of (7) is applied to derive the optimum transmit waveform spectrum which is shown in the bottom panel of Fig. 2. In this low-energy case, the optimized waveform is formed by the 'waterfilling action' by selecting two frequency bands of the target spectrum, i.e, the frequency bands with the largest coefficients. Looking at the optimum waveform spectrum, one can clearly see that the amplitudes are scaled in order to compensate for the clutter spectrum. Thus, in the case of a low-energy constraint, the optimum waveform only selects the dominant frequency components of the target spectrum. However, in the formation of the optimum waveform, the frequency coefficients are adjusted by the waterfilling solution in an attempt to compensate for the clutter spectrum.

With the same target, clutter, and noise, we apply the waterfilling action of (7) with a larger energy constraint of $E_x = 1.0$. The designed optimum transmit waveform spectrum is also shown in the bottom panel of Fig. 2. Notice that the optimum transmit waveform is very different from the optimum transmit waveform created with constraint $E_x = 0.01$. To maximize the mutual information between the target ensemble and received waveform, the high-energy transmit waveform distributes its constrained energy over a substantial portion of the target spectrum unlike the limited-energy case where only a few dominant frequency components are chosen. Even in the high-energy case, however, the optimum waveform spectrum emphasizes those frequencies where clutter power is lowest.

B. Target #2, Clutter #2, $E_x = 0.01, 0.1, 1.0$

In this example, we consider a different target spectrum labeled Target #2 and a different clutter spectrum, Clutter #2, which has a cosine-squared shape. These spectra are shown in the top panel of Fig. 3. The energy constraint is varied from $E_x = 0.01$, 0.1, and 1.0. The bottom panel of Fig. 3 shows the result of waterfilling solution with varying energy constraint. For the limited-energy constraint $E_x = 0.01$, the transmit waveform has selected a few frequency bands, which reaffirms the previous observation that for the low-energy constraint, the waterfilling action selects a few dominant frequency components and distributes among these components the available energy while compensating for clutter. For the intermediate energy constraint $E_x = 0.1$, the optimum waveform begins to utilize additional frequency components and fills them with the available energy while simultaneously "whitening" the clutter spectrum. For the relatively large energy constraint $E_x = 1.0$, notice that the outer frequency component of the optimum waveform had been amplified considerably higher than the rest of the frequency components of the target spectrum. The outer frequency band is located where clutter is least significant. Thus, for a high-energy constraint, the optimum waveform

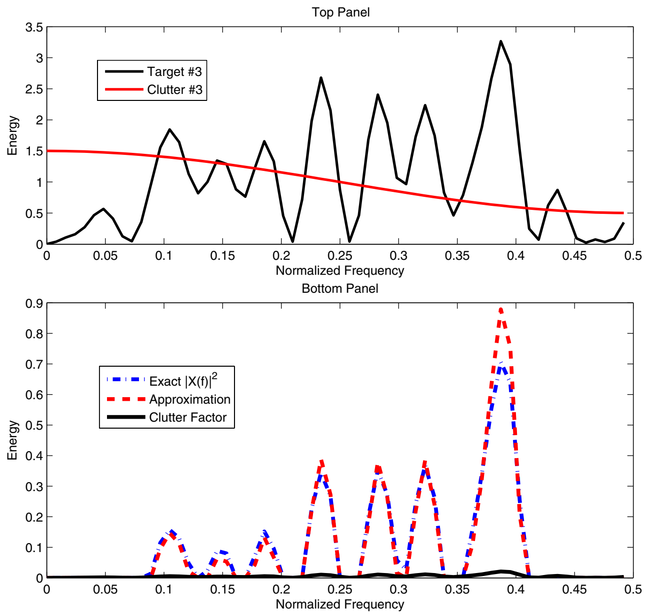

Fig. 4. Target #3, Clutter #3, Exact Waterfilling waveform, Approximation Waterfilling waveform and Clutter Factor $B(f)$

concentrates considerable energy to frequency components where signal-dependent interference is potentially lowest as a way of opportunistically taking advantage of the nulls of the clutter spectrum.

C. The Exact Solution, Approximation, and Clutter Factor

The first-order Taylor approximation of the optimum waveform was shown in (15). Furthermore, it was also commented that the clutter factor (14) has a major effect on the formation of the waterfilling waveform. We now look at the spectral shape of the exact waterfilling waveform, the energy spectrum of the Taylor approximation-based solution, and the clutter factor spectrum (14). For this design experiment, we use another arbitrary target spectrum labeled Target #3 and another low-pass shaped clutter spectrum labeled Clutter #3 shown in the top panel of Fig. 4. The bottom panel of Fig. 4 shows the exact solution derived from (7) with energy constraint of $E_x = 0.1$, the approximation solution derived from (15), and the clutter factor (14). There are two important points to observe. First, the approximate solution clearly resembles the exact solution and may be sufficient for specific resource-constrained applications. Note that the mathematical computations required by (13-15) are less intensive compared to (7-10). Second, the clutter factor clearly becomes a corrective measure that emphasizes or de-emphasizes the optimum waveform's frequency components according to a compromise among the target, clutter, and noise spectra.

V. Application to Cognitive Radar: Target Class Identification

A. Description of the Cognitive Radar

Ignoring the clutter signal block $c(t)$ connected by the dashed lines, Fig. 5 represents the closed-loop radar system in additive white Gaussian noise proposed for target recognition in [2]. In this figure, the CR signal processing involved is best described by a block labeled 'Cognitive Radar'. The next paragraph summarizes the operational framework of the radar system investigated in [2].

Consider a target identification problem in which one of M possible "known" targets is present. A Bayesian representation of the channel is formulated where the target hypotheses are denoted by $H_1, H_2, ..., H_M$ with corresponding prior probabilities P_1, P_1, ..., and P_M. Each hypothesis is described by a known finite time duration response $g_i(t), i = 1, 2, ...M$ with a corresponding energy spectrum $G_i(f), i = 1, 2, ...M$. In the noise-only case, the received echo is

$$y(t) = x(t) * g_i(t) + n(t), \qquad i \in 1, 2, ...M. \tag{16}$$

The first step for CR operation is to initially illuminate the channel with an optimum waveform based on the prior probabilities and energy spectra of the M hypotheses by forming a probability-weighted spectral variance

$$\sigma_H^2(f) = \sum_{i=1}^{m} P_i |G_i(f)|^2 - \left| \sum_{i=1}^{m} P_i G_i(f) \right|^2 \tag{17}$$

where the optimum waveform is dictated by the frequency-waterfilling waveform of (4). To facilitate computer simulation, discrete-time signal model is used where $T_s = 1$. Then, let $\mathbf{g}_i$ be the length-L target vector and $\mathbf{x}$ be the length-L_x transmit vector where

$$E_{\mathbf{x}} = \sum_{l=1}^{L_x} x^2[l] = \mathbf{x}^T \mathbf{x} \tag{18}$$

is the energy in $\mathbf{x}$. If we define a transmit signal convolution matrix of size $L_y \times L$ given by

$$\mathbf{X} = \begin{bmatrix} x(1) & 0 & \cdots & \cdots & 0 \\ x(2) & x(1) & \ddots & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ x(L_x) & x(L_x-1) & \cdots & x(1) & 0 \\ 0 & x(L_x) & x(L_x-1) & \cdots & x(1) \\ \vdots & 0 & x(L_x) & \cdots & x(2) \\ \vdots & \vdots & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & x(L_x) \end{bmatrix}, \tag{19}$$

then the length-L_y received signal vector is given by

$$\mathbf{y} = \mathbf{X}\mathbf{g}_i + \mathbf{n} \tag{20}$$

where L_y is given by $L_y = L_x + L - 1$.

The CR updates the prior probabilities by processing the received echo. A sequential probability ratio test (SPRT)

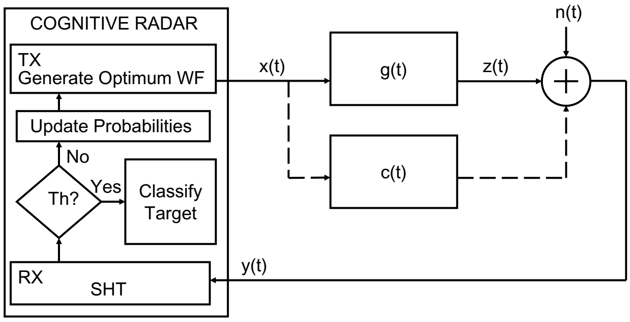

Fig. 5. Closed-loop radar system

[7,8] provides the criteria for making decisions after each illumination. In particular, the multihypothesis sequential test is applied where $\alpha_{i,j}$ for $i \neq j$ is the probability of incorrectly selecting H_j given that is H_i true. Let $\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_k$ be the received echoes of k successive transmissions, then the likelihood ratio for a pair of hypotheses i and j with prior probabilities P_i and P_j is

$$\Lambda_{i,j}^k = \frac{p_i(\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_k)}{p_j(\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_k)} \frac{P_i}{P_j} \tag{21}$$

where $p_i(\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_k)$ and $p_j(\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_k)$ are the joint pdfs under H_i and H_j hypotheses respectively. A decision is made for some m when the threshold

$$\Lambda_{m,j}^k > \frac{1 - \alpha_{m,j}}{\alpha_{m,j}} \tag{22}$$

is met. Then $K = k$ is the number of illumination cycles needed to make a target classification. If the threshold (22) is not met, the target probabilities are updated. For some target hypothesis H_i, the update rule is

$$P_i^{k+1} = \beta p_k(\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_k) P_i^k \tag{23}$$

where β ensures unity probability.

The application of information-based optimum waveforms in a clutter environment is seen by considering Fig. 5, where the clutter $c(t)$ block is activated. The random $c(t)$ is illuminated and received as a signal-dependent interference. When the target is random, $g(t)$ in Fig. 5 is now replaced by a Gaussian target ensemble where the target realization is unknown a priori. Thus, the application of the information-based waveform is a target class identification problem, i.e., the target realization to be identified belongs to one of the possible M target spectral variances described by $\sigma_1^2(f)$, $\sigma_2^2(f)$, $\ldots$, and $\sigma_M^2(f)$. A Bayesian representation of the channel is formulated, where the corresponding target class hypotheses and prior probabilities are denoted by $H_1, H_2, \ldots, H_M$ and $P_1, P_1, \ldots, P_M$. It is the job of CR to identify which ensemble class a stochastic target belongs to.

Utilizing the discrete-time signal model of (1), the length-L_y received signal vector $\mathbf{y}$ is given by

$$\mathbf{y} = \mathbf{g}_i * \mathbf{x} + \mathbf{c} * \mathbf{x} + \mathbf{n} \tag{24}$$

where the length-L vector $\mathbf{g}_i$ is a discrete target realization under the i^{th} hypothesis with target covariance $\mathbf{K}_{\mathbf{g}i}$ and $\mathbf{c}$ is the length-L clutter realization where the clutter covariance is denoted by $\mathbf{K_c}$. Using the convolution matrix formulation of (19), (24) may be represented by

$$\mathbf{y} = \mathbf{X}\mathbf{g}_i + \mathbf{X}\mathbf{c} + \mathbf{n}. \tag{25}$$

The resulting optimum waveform described by (7) is used to form the initial and successive probing waveforms via a probability-weighted spectral variance

$$\sigma_H^2(f) = \sum_{i=1}^m Pr(H_i)\sigma_i^2(f) - \left| \sum_{i=1}^m Pr(H_i)\sqrt{\sigma_i^2(f)} \right|^2. \tag{26}$$

SHT is used to update the channel probabilities where the threshold (22) is used by SPRT to terminate the experiment. In the case where both target and clutter realizations remain constant during the course of the identification, the joint pdf in (21) after the k^{th} observation is given by

$$p_i(\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_k) = \sqrt{\frac{|\mathbf{Q}^{-1}|}{(2\pi\sigma_n^2)^{kL_y}|\mathbf{K}_i|}} \exp\left[-\frac{1}{2\sigma_n^2} \sum_{j=1}^k \mathbf{y}_k^T \mathbf{y}_k \right] \times \exp\left[-\frac{1}{2\sigma_n^4} \left(\sum_{j=1}^k \mathbf{y}_k^T \mathbf{X}_k \right) \mathbf{Q}^{-1} \left(\sum_{j=1}^k \mathbf{y}_k^T \mathbf{X}_k \right) \right] \tag{27}$$

where $\mathbf{X}_k$ is the convolution matrix of the transmit waveform and $\mathbf{K}_i$ and $\mathbf{Q}$ are defined by the following equations

$$\mathbf{K}_i = \mathbf{K}_{\mathbf{g}_i} + \mathbf{K_c} \tag{28}$$

$$\mathbf{Q} = \mathbf{K}_i^{-1} + \frac{1}{\sigma_n^2} \sum_{j=1}^k \mathbf{X}_k^T \mathbf{X}_k. \tag{29}$$

B. Results of CR Application

One application of cognitive radar is to efficiently classify targets as quickly as possible, i.e., given a waveform energy constraint, the number of transmissions K it takes to classify a target should be minimized. Since the noise, target, and clutter are random, K is clearly random. Specifically, we are interested in the average value of K, also called the average sample number (ASN). We estimate this moment by using Monte Carlo simulation. We investigate the performance of the optimum MI-based waveform in signal-dependent interference. The experimental setup is described in the next paragraph.

First, four minimally overlapped spectral variances that correspond to M = 4 target ensemble classes are generated. The misclassification rate is set to 0.01. A low-pass shaped PSD is used to model clutter. Then, a target class is randomly chosen. A particular realization of this target ensemble and a clutter realization both of vector length $L = 31$ are generated. With the target and clutter signals fixed, 100 Monte Carlo trials are performed. This experiment is performed over 1000 realizations of the target and clutter for each energy level. ASN is then calculated for each of those energy levels. Three

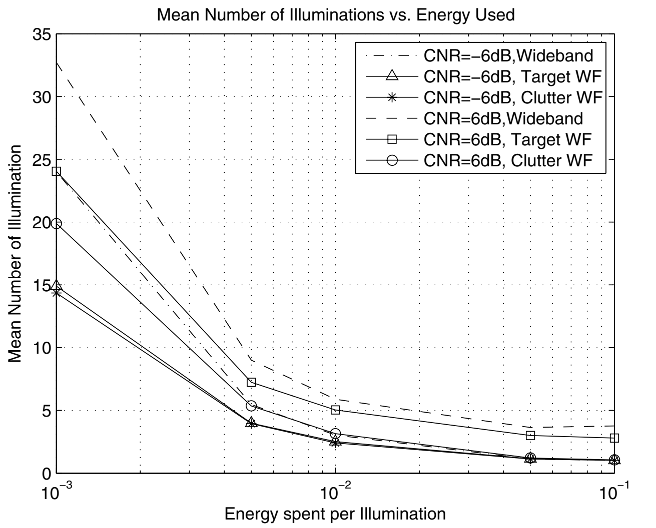

Fig. 6. Performance comparison of Wideband pulse, Target WF, and Clutter WF

transmit waveform types are used; a discrete-time impulse waveform denoted by 'Wideband', the noise-only waterfilling waveform denoted by 'Target WF', and the optimum waveform in signal-dependent interference denoted by 'Clutter WF'. In this experiment, the target-to-noise ratio (TNR) is set at 15dB while the clutter-to-noise (CNR) ratio is set to -6dB and 6dB. Fig. 6 shows the performances of the three waveforms described above. For the two cases of CNR -6dB and 6dB, the 'Clutter WF' performed the best, i.e., the optimum waveform tailored for signal-dependent interference has the lowest mean number of iterations at any energy constraint. Note the noise-limited versus clutter-limited behavior in Fig. 6. At low transmit energy and low CNR, the received signal is noise limited. Hence, the noise-only 'Target WF' and the 'Clutter WF' perform nearly the same because clutter is not much of a factor. At high transmit energy and high CNR, the signal-dependent clutter is the dominant interference rather than the noise. Hence, the 'Target WF' and 'Clutter WF' behave differently. The minimum number of transmissions that can occur before making a decision is one. At low CNR, both waterfilling waveforms reach the minimum before clutter becomes a significant factor. In other words, at CNR = -6 dB, the received signal is never clutter limited even at higher transmit energy.

VI. Summary and Conclusion

The information-based approach to waveform design for signal-dependent interference and channel noise was investigated. The mutual information between the target ensemble and received signal where the transmit waveform has an energy constraint yielded a waterfilling solution in the frequency domain. To maximize mutual information, a radar system must customize the transmit waveform in such a way to de-emphasize frequency bands where clutter is significant and emphasize frequency bands where clutter is negligible. While this behavior is common to all the energy constraints, there is a definite and interesting pattern in the selection of frequency bands to be utilized and filled as the energy constraint is increased. In particular, for a low energy constraint, the optimum waveform only selects the target spectrum's frequency components with the largest coefficients. For large energy constraint, the optimum waveform opportunistically utilizes the target frequency components located at clutter nulls and fills them with considerable amount of the available energy while other frequency components are also opportunistically filled as dictated by the compromise among target, clutter, and noise spectra.

This optimum waveform was applied in a CR setting where the target is buried in clutter environment. Simulations show that the proposed waveform performs well in terms of the lowest mean number of iterations to arrive at the correct target class compared to a wideband impulse waveform and the waterfilling waveform where clutter is unaccounted for.

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