Evaluation of Modulus-Constrained Matched Illumination Waveforms for Target Identification

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Abstract—**In prior work, we have applied matched illumination strategies to target identification by a closed-loop radar system. In the closed-loop system, multiple waveforms are transmitted in succession, but each is customized based on the returns from prior transmissions. In this prior work, however, the matched waveforms were not constrained to be constant modulus. This current paper evaluates the performance of closed-loop radar with constant-modulus matched illumination. We also compare the performance of non-constant-modulus illumination under a peak power constraint. Finally, we use simple target models and assume unknown orientation, rather than the deterministic or Gaussian target models used in earlier work.**

I. INTRODUCTION

While many modern radars focus on improving performance by adaptive signal processing in the receiver, cognitive radar [1] focuses on improving performance not only by adaptive signal processing, but also through an adaptive radar transmitter. One example of cognitive radar that we have presented is where the adaptive transmitter generates a new temporal waveform – optimized for target identification – based on prior measurements and prior knowledge. Thus, the cognitive radar encompasses the transmitter, receiver, and environment interacting together in a closed-loop system. Our earlier results show improved performance in resource-constrained (i.e., finite energy) and interference-limited environments.

A high-resolution range profile (HRRP) is a radar target signature that is backscattering power recorded as a function of delay when a wideband signal is transmitted [2]. However, even a slight change in target aspect angle generates a different HRRP, so many HRRPs must be stored in a template library in order to accurately represent the target. This paper builds on earlier work in the following ways. First, in earlier work the targets have been modeled as either deterministic or Gaussian with a known PSD. In practice, targets are neither, so here we have implemented very simple scattering models that vary with angle. The target orientation is unknown, but assumed to lie within a certain range. Second, we apply constant modulus constraints to the matched illumination waveform designed by the mutual information metric. Thus, the goal of this paper is to implement closed-loop, matched illumination with more realistic target models and waveform constraints.

The outline of this paper is as follows. In Section II, we develop the problem statement and signal model. In Section III, we present the target model. In Section IV, we briefly show the waveform design technique. In Section V, we describe how to generate the constant modulus waveform from a given Fourier transform magnitude. We also discuss the peak-power (peak modulus) waveform normalization. In Section VI, we review the decision-making procedure and probability updates for closed-loop radar. In Section VII, we present simulated results, and in Section VIII, we make conclusions.

II. PROBLEM STATEMENT AND SIGNAL MODEL

The target discrimination performance of monostatic radar with customized waveforms is evaluated in the presence of additive white Gaussian noise (AWGN). The problem formulation is as follows. It is assumed that one of M targets is present. Target signatures are sensitive to target aspect angle, so the set of target profiles are divided into uniformly spaced sectors. The number of sectors is assumed to be N_h for each target. Target profiles are then calculated at multiple angles within each sector, and the profiles are averaged to obtain a mean template for that sector [3]. The target profiles for each target are assumed to be known exactly for a given angle; therefore, the mean templates for the N_h sectors for all M targets $h_i(t), i = 1,..., N_h,..., MN_h$ are also perfectly known. The exact target orientation is not known, but assumed to lie within just a few sectors. This represents prior knowledge about the approximate target orientation. The true target signature, $g(t)$, is drawn from a random angle within the range of angles specified by the prior knowledge. We want to determine not only which target $g(t)$ belongs to, but also which angular sector. The radar waveform is defined as $s(t)$, and the received signal $y(t)$ due to $s(t)$ is defined as

$$y(t) = g(t) * s(t) + n(t) \quad (1)$$

where * represents the convolution operator and $n(t)$ indicates AWGN that has normalized power $\sigma_n^2 = 1$.

We consider the probability density function (pdf) of $y(t)$ under the i^{th} hypothesis. We treat $h_i(t)$ as deterministic and since $n(t)$ is AWGN, the distribution of $y(t)$ under the i^{th} hypothesis is Gaussian. Switching to a discrete-time notation [4], the received signal is $\mathbf{y} = \mathbf{Sg} + \mathbf{n}$ [4], and the pdf of $\mathbf{y}$ under the i^{th} hypothesis is

$$p\left(\mathbf{y} \mid H_i\right) = \frac{1}{(\pi)^N \mid \mathbf{K}_{y,i} \mid} \exp(-(\mathbf{y} - \boldsymbol{\mu}_{y,i})^H \mathbf{K}_{y,i}^{-1} (\mathbf{y} - \boldsymbol{\mu}_{y,i})) \quad (2)$$

where $\mathbf{K}_{y,i} = \sigma_n^2 \mathbf{I}$ is the covariance matrix of $\mathbf{y}$ under i^{th} target hypothesis and $\boldsymbol{\mu}_{y,i} = \mathbf{Sh}_i$ is the waveform-dependent mean of $\mathbf{y}$ under the i^{th} hypothesis. In [5] and other techniques, classification is based on the estimated target HRRP, so to use this method we would need to estimate the target HRRP from the measured signal $\mathbf{y}$.

If the transmitted waveform is a high-bandwidth waveform with a "good" (i.e., low-sidelobe) autocorrelation function, then all that needs to be done is to apply matched filtering. However, the matched illumination waveforms that we calculate are designed to classify targets, not necessarily to generate a quality estimate of the target's range profile. In other words, a "good" waveform for target identification does not necessarily produce a good image of the target. The reason that typical classification techniques use estimated HRRP's is that HRRP's are naturally intuitive for humans to understand and to build algorithms around, not because they have inherent advantages in classification. Thus, we do not estimate the HRRP here because the waveform we use often has a poor autocorrelation function, and the resulting estimated HRRP does not give a good result. Instead of estimating the HRRP, every time we update the transmitted waveform, we update the signal matrix $\mathbf{S}$ in the calculation of the mean $\boldsymbol{\mu}_{y,i}$.

III. TARGET MODEL

The simplest way to model a target is using a point target model, in which case the target's signature is just an impulse with the proper delay, phase and amplitude. However, this model does not accurately reflect what can be observed by a radar when resolution is sufficient for resolving many scattering centers on the physical target. A more realistic model is to consider targets such as planes and ships to have many scattering points, each modeled as a point target. Then, a target impulse response can be modeled according to

$$h(t) = \sum_{i=1}^{N_s} \gamma_i \delta(t - \tau_i) \quad (3)$$

where N_s is the number of scattering centers on the target, γ_i is the scattering coefficient of the i^{th} scattering center, $\delta(t)$ is the Dirac delta function, and τ_i is the round trip time for the transmitted waveform to travel from the radar to the target and back. The impulse function in (3) has infinite bandwidth in the frequency domain and cannot be modeled directly in a computer simulation. However, any observed radar signal will be bandlimited, so it is possible to represent the target with a the bandlimited transfer function $H(w)$ that equals

$$H(w) = \sum_{i=1}^{N_s} \gamma_i e^{-jw\tau_i} \quad (4)$$

within the radar bandwidth and zero elsewhere. We can model the transfer function according to (4) over the radar band and convert back to the time domain via inverse Fourier Transform to model the target impulse response. The time-domain impulse response is now time-limited and can be represented with a finite number of samples.

To make simple targets, we define our own arbitrary target outlines. The targets have physical size that is generally accurate for targets of interest, and we place scatters in key positions along the target outline. As we change aspect angle, the targets are rotated as well as the scatters. Thus, the relative range between radar and the scatters changes, causing fluctuations due to fading, and hence changes in the target impulse response. The target outlines are admittedly low-fidelity at this point; ongoing work is being done to incorporate target models calculated using high-fidelity electromagnetic (EM) software.

IV. WAVEFORM DESIGN

The waveform design strategy that we use is based on mutual information. The strategy was originally presented in [6] by Bell and is summarized as follows. It is assumed that we have an ensemble of target impulse responses. If we assume that the transmit waveform is time-, energy-, and approximately frequency- limited, the transmit waveform that maximizes the mutual information between the ensemble of impulse responses and the radar received signal exists, and the waveform has an energy spectrum according to

$$|S(f)|^2 = \begin{cases} \max\left[0, A - \dfrac{\sigma_n^2 T_y}{2\sigma_H^2(f)}\right] & |f| \le \dfrac{1}{2T_s} \\ 0 & |f| > \dfrac{1}{2T_s} \end{cases} \quad (5)$$

where the ensemble's spectral function is denoted as $\sigma_H^2(f) = E\{|H(f) - E\{H(f)\}|^2\}$, $H(f)$ is the target transfer function, and T_s the interval at which the waveform and impulse responses are sampled. The energy constraint A is calculated according to the equation

$$E = \int_{-1/2T_S}^{1/2T_S} \max\left[0, A - \frac{\sigma_n^2 T_y}{2\sigma_H^2(f)}\right] df . \quad (6)$$

The waveform design technique above is for a Gaussian ensemble, but for a finite number of hypotheses in a target identification scenario, the ensemble of potential transfer functions is not Gaussian. However, the procedure is still intuitively correct, and we extend the spectral variance function according to [7]

$$\sigma_H^2(f) = \sum_{i=1}^{MN_h} P_i |H_i(f)|^2 - |\sum_{i=1}^{MN_h} P_i H_i(f)|^2 \quad (7)$$

where $H_i(f)$ is the transfer function associated with the *i*th mean template. This technique generates the waveform by pouring energy into the function $\sigma_n^2 T_y / 2\sigma_H^2(f)$ until the energy constraint is met. This technique is called waterfilling [6][8].

V. New Waveform Constraints

Bell applied energy, time, and frequency constraints to generate the information-based waveform in [6], but here we add two more constraints, which are constant modulus [9][10] and maximum modulus normalization [10][11] constraints. The two constraints are desired because we do not want the temporal waveform to have high peak amplitude and to operate inefficiently by operating during part of the waveform at less than peak power.

The technique to generate a constant modulus signal from a given Fourier transform is summarized below and based on the technique in [9]. We define D_M as the group of functions $\{v(t)\}$ that have the same Fourier transform magnitude $F(w)$ over the frequency interval Ψ. Then, an arbitrary function $x(t)$ can be projected to a closest point on D_M by a magnitude projection operator P_M. When the Fourier transform of $x(t)$ can be represented by $X(w) = |X(w)| e^{j\Omega(w)}$, the magnitude projection of an arbitrary function $x(t)$ is defined as

$$P_M x(t) = \begin{cases} F(w) e^{j\Omega(w)}, & w \in \Psi \\ X(w), & w \in \Psi' \end{cases} . \quad (8)$$

We also denote D_A as the set of functions $\{v(t)\}$ that have constant amplitude B over the time interval T. Then, an arbitrary function $x(t)$ can be projected to a nearest point on D_A by an amplitude projection operator P_A, and the projection procedure is

$$P_A x(t) = \begin{cases} B e^{j\phi(t)}, & t \in T \\ x(t), & otherwise \end{cases} \quad (9)$$

where $x(t)$ can be represented by $a(t)e^{j\phi(t)}$. The magnitude and amplitude projection are combined according to

$$x_{k+1}(t) = P_A P_M x_k(t) \quad (10)$$

where $x_k(t)$ is the k^{th} projection iterated function. After a number of magnitude and amplitude projections, the function $x(t)$ has constant modulus envelope while approximately maintaining the prescribed Fourier transform magnitude.

Another constraint is the maximum modulus normalization presented in [10][11], which is essentially a constraint on the instantaneous peak power. When applying this constraint, we force the signal envelope to have maximum amplitude of

$$A = \sqrt{E_s / N_w} \quad (11)$$

where E_s the waveform energy and N_w is the number of discrete-time samples representing the waveform. We can apply this normalization to any of the waveforms we study, but because the constraint is essentially a scaling factor, the shape of the waveform does not change. If we design a non-constant modulus waveform with energy E_s, but then apply the maximum modulus constraint, the resulting waveform will have less energy than originally intended.

VI. Fixed Number of Iterations and Bayes' Theorem

To evaluate waveform performance, we perform experiments where the radar makes a predetermined, fixed number of transmissions before making a decision. After every observation, we compute the likelihoods for each hypothesis. These likelihoods are used to update the waveform, but a decision is only made after the predetermined number of observations. The likelihood expression of the i^{th} hypothesis after the k^{th} illumination can be formed according to

$$\mathbf{\Lambda}_i^k = p_{i1}(\mathbf{y}_1) p_{i2}(\mathbf{y}_2) \cdots p_{ik}(\mathbf{y}_k) P_i \quad (12)$$

where $p_{ik}(\mathbf{y}_k)$ is the pdf of the received signal due to the k^{th} transmitted waveform for the i^{th} hypothesis (defined above in (2)), $\mathbf{y}_k$ is the signal received on the k^{th} transmission, and P_i is the prior probability of the i^{th} hypothesis before taking any measurements. After the given number of observation, the experiment is terminated. The decision H_i is made in favor of the target and sector with the highest likelihood.

At each transmission, we update the hypothesis probabilities to modify the waveform. Using Bayes'

Theorem, the hypothesis probability after the k^{th} transmission is

$$p(H_i \mid \mathbf{y}_k) = \frac{\Lambda_i^k}{p(\mathbf{y}_k)} \tag{13}$$

where $p(\mathbf{y}_k)$ is a scaling factor such that the sum of the $p(H_i \mid \mathbf{y}_k)$ is one. The waveform is modified according to updated hypothesis probabilities as described in [7].

VII. RESULTS

We apply the theory we have discussed to a computer simulation. We assume that we have two targets. The orientation for each target is divided into uniformly spaced sectors. The sectors are one degree wide, and the library is defined at 0.2-degree intervals. The target HRRPs within a sector are averaged to obtain a mean template for use in the probability updates and waveform calculations. Thus, we have a mean template for every one degree. We randomly choose a target and orientation angle and use two adjacent templates for each of the two targets to form four hypotheses.

We compare the performance of four different waveforms in identifying these hypotheses. The first three waveforms are information-based waveforms [6], and the last is a wideband waveform. The first information-based waveform is obtained directly from waterfilling in the frequency domain. The time-domain waveform is found via inverse DFT and results in a non-constant modulus temporal shape. The second information-based waveform is a constant-modulus version [9] that approximates the optimum mutual-information-based spectrum. The third information-based waveform is a scaled version of the non-constant modulus waveform, scaled to meet the maximum modulus constraint [10][11]. We perform five transmissions to make a decision. We run 10,000 Monte Carlo trials to estimate the probability of an incorrect decision.

Figure 1 shows the complex constellation of temporal waveform before and after the constant modulus constraint is applied. Before the constant modulus constraint is applied (the unconstrained optimized waveform), the temporal waveform has fluctuation amplitudes and also high peak amplitudes. However, after the constant modulus constraint is applied (the optimized waveform with the constant modulus constraint), the temporal waveform has constant amplitude. Figures 2 and 3 show the corresponding Fourier transform magnitudes. Figure 2 shows the Fourier transform magnitude of the non-constant modulus optimized waveform. Figure 3 shows the Fourier transform magnitude of the optimized waveform with the constant modulus constraint. The constant modulus constraint spreads the waveform energy into additional frequency bands, but the two high peak magnitudes are maintained. The two Fourier transform magnitudes look similar.

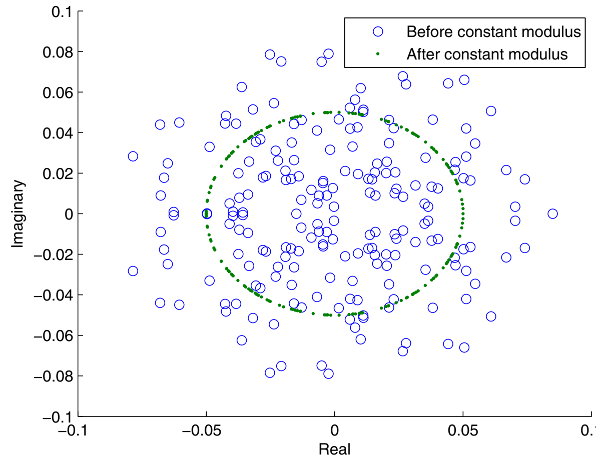

Figure 1. Complex constellation of unconstrained and constant modulus constrained waveform.

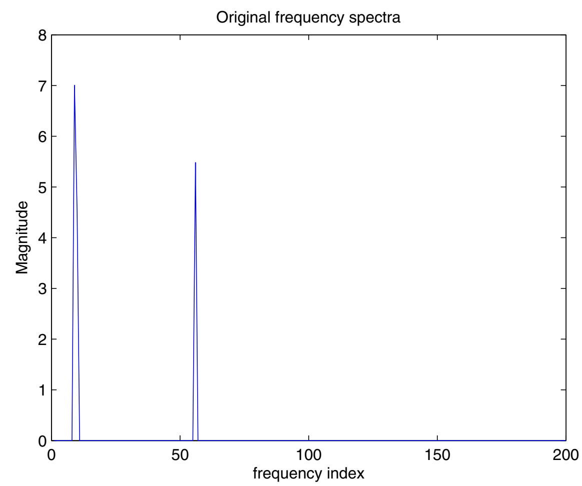

Figure 2. Fourier transform magnitude of unconstrained waveform.

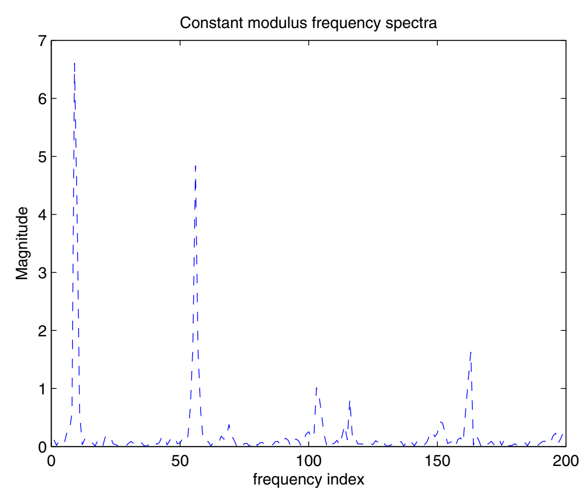

Figure 3. Fourier transform magnitude of constant modulus constrained waveform.

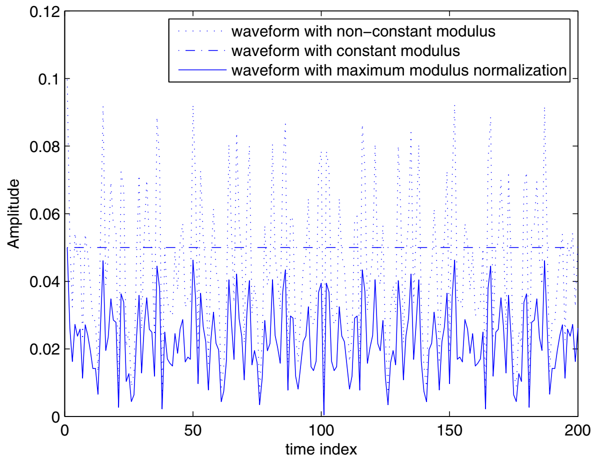

Figure 4. The temporal structure of three mutual information based waveform.

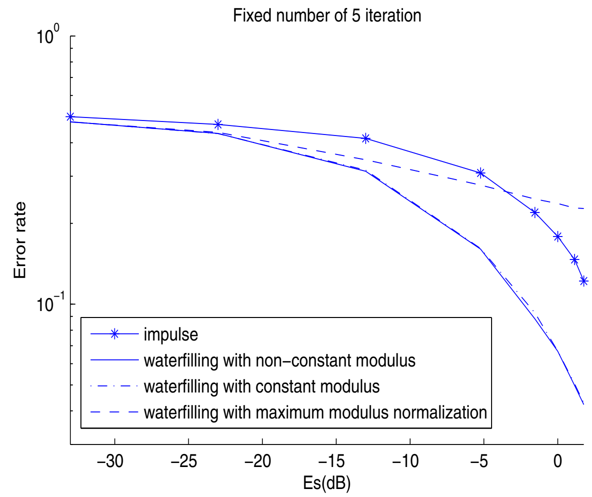

Figure 5. Error rates versus waveform energy in fixed number of iterations.

Figure 4 shows the temporal structure of all three mutual-information-based waveforms. The non-constant modulus waveform has amplitude that varies, but the constant modulus waveform has constant amplitude. The maximum modulus normalization waveform looks like the non-constant modulus waveform, but the peak is matched to the constant modulus waveform. Figure 5 shows error rates of the target discrimination. We apply new constraints for constant modulus and maximum modulus normalization waveforms, respectively. The result shows that the unconstrained optimized waveform performs best, but the performance loss is minor for the sub-optimum waveform with the constant modulus constraint. The reason is that the Fourier transform magnitude of the sub-optimum waveform with the constant modulus constraint has similar Fourier transform magnitude as the unconstrained optimized waveform. The performance loss for maximum modulus normalization becomes high as the waveform energy increases, because the waveform with maximum normalization constraint does not produce as much energy in the allotted time.

VIII. CONCLUSIONS

We have implemented a closed-loop radar system with two new constrained adaptive waveforms for target discrimination. The constraints are constant modulus [9] and maximum modulus normalization [10][11]. The waveforms were implemented in the radar system such that the temporal waveforms are modified based on previous measurements and prior information. We also used more realistic target models than used in previous work. The result shows that the constant modulus waveform has only a small performance loss compared to the optimized non-constant modulus waveform. The maximum modulus normalization shows high error rate as the waveform energy increases because this waveform operates inefficiently.

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