Detection Performance of Multibranch and Multichannel Compressive Receivers

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Abstract—**Some proposed architectures for compressive sampling of time-domain signals involve splitting the signal into multiple branches. This approach is not unlike the idea of splitting a signal into in-phase and quadrature branches for a coherent receiver implementation, but in the case of a compressive receiver the idea is to modulate the signal in each branch against a different measurement kernel. Therefore, multiple projections can be captured in parallel over the same time interval of the signal.**

In this paper, we evaluate the detection performance of multibranch compressive receiver implementations. The noise statistics over receiver branches are derived and shown to depend on the quality of the low-noise amplifier (LNA). A high-gain, low-noise-figure amplifier causes correlated noise across the receiver branches, but avoids SNR loss due to the power divider. We discuss whether a multibranch approach can provide equal or better detection performance than a single-branch compressive receiver, and include comparisons for varying LNA quality. A comparison to a true multichannel implementation is also provided.

I. INTRODUCTION

Compressive sampling offers the opportunity to reconstruct certain signals that have been acquired at sampling rates below the Nyquist rate. In order to accomplish this undersampling, the signal must be modulated against a measurement kernel before integrating the result to obtain a measurement. The random demodulator [1] and modulated wideband converter [2] are two architectures for direct modulation and compressive sampling of sparse signals.

These receivers are generally intended to operate in the high-SNR regime where sparse reconstruction algorithms can be used to reconstruct the original signal. In radar, however, we are often interested in detecting signals that are buried in noise before increasing the signal-to-noise ratio (SNR) through matched filtering. On the other hand, compressed sensing (CS) exploits measurements that are unmatched, or incoherent, which results in signal energy loss [3] or, equivalently, noise folding [4-5]. As compressed sensing techniques find their way into radar applications, it is worthwhile to consider the detection performance loss incurred by compressive sampling. Earlier work on this topic was presented in [3]. In the current paper, we consider a general multibranch receiver architecture that has the receivers in [1] and [2] as particular cases. We show that in addition to SNR loss incurred due to reduced signal energy, multibranch receivers will suffer from degradation that varies with the number of branches and the quality of the low-noise amplifier (LNA) that precedes the signal power divider.

We describe a general compressive receiver architecture in Section II and evaluate the noise figure and noise correlation for this architecture in Section III. In Section IV, we look at the corresponding structure of the CS measurement matrix. Detection performance is evaluated via simulation in Section V, and conclusions are made in Section VI.

II. MULTIBRANCH RECEIVER ARCHITECTURE

Consider the general structure of a compressive receiver shown in Figure 1. A noisy lowpass signal $\tilde{x}(t)=s(t)+\tilde{n}(t)$ enters the compressive receiver at left. The signal is assumed to have single-sided (lowpass) bandwidth $B/2$, and the noise is Gaussian distributed and white (over a bandwidth much larger than the signal bandwidth) with average power P_i. The noisy signal passes through an LNA having gain G_a and noise figure F_a, and is then split into L branches by a power divider. The signal on the lth branch is then modulated with the measurement kernel, $k_l(t)$, passed through a lowpass filter (LPF) or ideal integrator, and finally sampled by an analog-to-digital converter (ADC) to complete the measurement projection. If each branch of the receiver is sampled at a rate of F_{ADC}, then the total effective sampling rate of the receiver is $\tilde{F}_s = LF_{ADC}$.

Assuming for a moment that the LNA has an ideal noise figure of $F_a = 1$ and that $G_a >> L$, the mth measurement on the lth branch of the receiver can be written as

$$y_l[m]=y_l(mT)=\int_{-\infty}^{\infty}\sqrt{G_a/L}\,\tilde{x}(t)k_l(t)h(mT-t)\,dt, \quad (1)$$

where $T = 1/F_{ADC}$ and $h(t)$ is the LPF's impulse response. Therefore, each measurement is a projection of the signal onto the measurement kernel over a unique window equal to

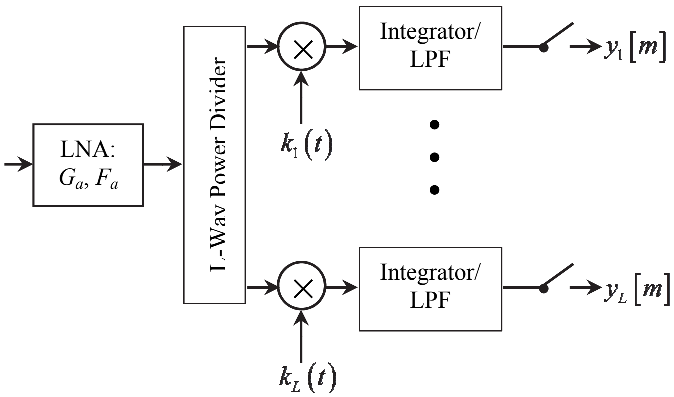

Figure 1. Multibranch CS receiver architecture.

a time-shifted copy of $h(t)$. For example, if we set

$$h(t) = \begin{cases} 1 & 0 \le t < T \\ 0 & \text{else} \end{cases},$$

then the LPF becomes an ideal integrator, and

$$y_l[m] = y_l(mT) = \int_{(m-1)T}^{mT} \sqrt{G_a/L}\, \tilde{x}(t)\, k_l(t)\, dt\,. \tag{2}$$

This general structure can model many of the compressive sampling or analog-to-information structures previously presented in the literature [1-2], and indeed is very similar to [2] except for explicit statements on the measurement kernels and filter functions. The structure can also represent a traditional sampler by setting the measurement kernel to

$$k_l(t) = \sum_m \delta(t - mT)\,,$$

such that the sifting property of the delta function gives

$$y_l[m] = \sqrt{G_a/L}\, \tilde{x}(mT)\,.$$

The reason for considering the multibranch structure in Figure 1 is that a single branch enforces a strict sequential structure on the sampling mathematics. Therefore, a single-branch receiver does not enable parallel measurements over the same time interval of the signal. In contrast, hardware-independent formulations of CS often assume that every sample can be obtained by a mixing of the input signal over the signal's entire duration, rather than over finite durations with fixed offset. It is interesting to consider whether the parallel structure can provide some benefit to signal detection, especially when the input signal has some unknown parameter such as an unknown time of arrival.

III. Noise Figure and Correlation Analysis

The detection performance of the multichannel receiver will be strongly affected by the specifications of the LNA. If the LNA is not very good, then the SNR loss introduced by the splitter will adversely impact signal detection. In this section, we derive the noise properties at the output of the splitter, including correlation across branches of the receiver.

Let us define the signal on the lth branch immediately after the L-way power divider as $x_l(t)$, which can be expressed as

$$x_l(t) = \sqrt{\frac{G_a}{L}}\, s(t) + n_l(t)\,. \tag{3}$$

The noise, $n_l(t)$, at this point in the receiver is the result of three contributors: the input noise that has propagated to the lth branch, the noise added by the LNA that has propagated to the lth branch, and the noise added by the power divider. Let $\hat{n}_a(t)$ be the additional noise contributed by the LNA and $\hat{n}_j(t)$ be the additional noise contributed by the power divider in the lth branch. Recalling the input noise $\tilde{n}(t)$, the total noise in the lth branch is

$$n_l(t) = \sqrt{\frac{G_a}{L}}\, \tilde{n}(t) + \sqrt{\frac{1}{L}}\, \hat{n}_a(t) + \hat{n}_l(t)\,. \tag{4}$$

The noise added by the LNA can be expressed using its noise figure and gain according to

$$P_{\hat{a}} = \mathrm{E}\left[\left| \hat{n}_a(t) \right|^2 \right] = (F_a - 1)\, G_a P_i\,. \tag{5}$$

Likewise, the noise added by the power divider can be expressed as

$$P_{\hat{l}} = \mathrm{E}\left[\left| \hat{n}_l(t) \right|^2 \right] = (F_d - 1)\, G_d P_i = \frac{L-1}{L} P_i \tag{6}$$

where we have used the fact that an ideal power divider's noise figure is $F_d = L$ and its gain is $F_d = 1/L$. Combining these noise powers, the total noise power on the lth receiver branch is

$$\begin{aligned} P_l = \mathrm{E}\left[\left| n_l(t) \right|^2 \right] &= \left(\frac{G_a}{L} + (F_a - 1)\frac{G_a}{L} + \frac{L-1}{L} \right) P_i \\ &= \left(\frac{F_a G_a}{L} + \frac{L-1}{L} \right) P_i \end{aligned}\,. \tag{7}$$

Because some of this noise originates before the power divider and some originates after the power divider, the noise will, in general, be partially correlated over the different branches. To derive this correlation, we begin by defining a noise autocorrelation function over temporal lag τ and branch offset p as

$$\begin{aligned} R_n(\tau, p) &= \mathrm{E}\left[n_l(t)\, n_{l+p}(t+\tau) \right] \\ &= \mathrm{E}\left[\begin{gathered} \left(\sqrt{\frac{G_a}{L}}\tilde{n}(t) + \sqrt{\frac{1}{L}}\hat{n}_a(t) + \hat{n}_l(t) \right) \times \\ \left(\sqrt{\frac{G_a}{L}}\tilde{n}(t+\tau) + \sqrt{\frac{1}{L}}\hat{n}_a(t+\tau) + \hat{n}_{l+p}(t+\tau) \right) \end{gathered} \right] \end{aligned}. \tag{8}$$

Assuming that noise originating from different components is uncorrelated, the cross terms in this expected value go to zero, and the autocorrelation function reduces to

$$
\begin{aligned}
R_n(\tau,p) &= \mathrm{E}\left[\left(\frac{G_a}{L}\tilde{n}(t)\tilde{n}(t+\tau)+\frac{1}{L}\hat{n}_a(t)\hat{n}_a(t+\tau)\right)\right] \\
&\quad +\mathrm{E}\left[\left(\hat{n}_l(t)\hat{n}_{l+p}(t+\tau)\right)\right] \\
&= \left(\frac{G_a}{L}+(F_a-1)\frac{G_a}{L}+\frac{L-1}{L}\delta[p]\right)P_i\,\delta(\tau) \\
&= \left(\frac{F_a G_a}{L}+\frac{L-1}{L}\delta[p]\right)P_i\,\delta(\tau)
\end{aligned} \quad . \tag{9}
$$

We see in (9) that the contributions due to the input noise and LNA will be correlated across branches while the noise due to the power divider will be uncorrelated across branches.

IV. Simulation Model and Measurement Matrix

For simulation purposes, the input analog signal is modeled in discrete time at a sample rate equal to or higher than the Nyquist rate. Defining the simulation sample rate as $F_{SN} >> B$, we represent the input signal $s(t)$ with a length-N input signal vector $\mathbf{s}$. The discrete-time signal on each receiver branch prior to multiplying with the compression kernel is

$$
\mathbf{x}_l = \sqrt{\frac{G_a}{L}}\mathbf{s}+\mathbf{n}_l \,. \tag{10}
$$

After passing through the multiplier and integrator/LPF, the temporally sampled data vector on the lth branch is

$$
\mathbf{y}_l = \mathbf{K}_l\mathbf{x}_l = \mathbf{K}_l\left(\sqrt{\frac{G_a}{L}}\mathbf{s}+\mathbf{n}_l\right). \tag{11}
$$

where the $M\times N$ measurement matrix $\mathbf{K}_l$ captures the effects of both the multiplication with $k_l(t)$ and the time-shifted filter impulse response. Defining $\mathbf{k}_l$ as the length-N Nyquist-rate representation of the kernel $k_l(t)$ and $\mathbf{h}_m$ as the Nyquist-rate representation of $h(mT-t)$, we can write

$$
\mathbf{K}_l = \begin{bmatrix}\mathbf{k}_l \\ \mathbf{k}_l \\ \vdots \\ \mathbf{k}_l\end{bmatrix} \odot \begin{bmatrix}\mathbf{h}_0 \\ \mathbf{h}_1 \\ \vdots \\ \mathbf{h}_{M-1}\end{bmatrix} \tag{12}
$$

where '$\odot$' denotes a Hadamard product. Figure 2 shows the typical structure of the envelope for this measurement matrix (the second term on the right-hand side of (12)) without the underlying modulation of the measurement kernel.

The structure in Figure 2 is repeated for each receiver branch, but modulated by the appropriate kernel $k_l(t)$. Defining the noisy multibranch signal at the output of the power divider as

$$
\begin{aligned}
\mathbf{x} &= \begin{bmatrix}\mathbf{x}_1^{\mathrm{T}} & \mathbf{x}_2^{\mathrm{T}} & \cdots & \mathbf{x}_L^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}} \\
&= \sqrt{\frac{G_a}{L}}\begin{bmatrix}\mathbf{s}^{\mathrm{T}} & \mathbf{s}^{\mathrm{T}} & \cdots & \mathbf{s}^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}} + \begin{bmatrix}\mathbf{n}_1^{\mathrm{T}} & \mathbf{n}_2^{\mathrm{T}} & \cdots & \mathbf{n}_L^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}
\end{aligned} , \tag{13}
$$

we can express the measurements as

$$
\mathbf{y} = \mathbf{K}\mathbf{x} \tag{14}
$$

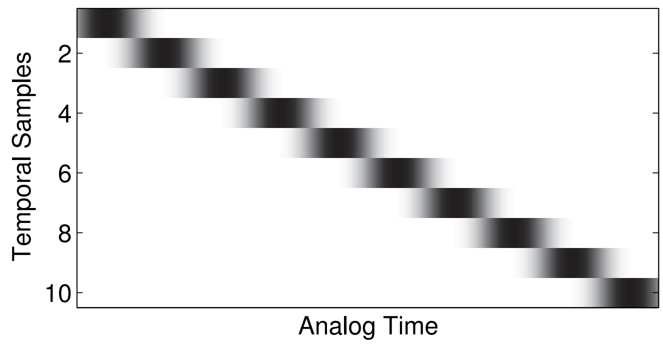

Figure 2. Envelope structure of single-branch CS receiver.

where $\mathbf{K}$ is the multibranch measurement matrix with block diagonal structure formed from the individual $\mathbf{K}_l$'s. Using the noise correlation properties from (9), the noise covariance matrix prior to compression is

$$
\mathbf{R}_n = \mathrm{E}\left[\mathbf{n}\mathbf{n}^{\mathrm{T}}\right] = P_i\left(\frac{F_a G_a}{L}\mathbf{1}_L \otimes \mathbf{I}_M + \frac{L-1}{L}\mathbf{I}_{LM}\right) \tag{15}
$$

where $\mathbf{1}_L$ denotes a length-L column vector of all ones, $\mathbf{I}_M$ denotes an $M\times M$ identity matrix, '$\otimes$' denotes a Kronecker product, and $\mathbf{n}$ is defined as

$$
\mathbf{n} = \begin{bmatrix}\mathbf{n}_1^{\mathrm{T}} & \mathbf{n}_2^{\mathrm{T}} & \cdots & \mathbf{n}_L^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}. \tag{16}
$$

The noise covariance matrix after compression is

$$
\mathbf{R}_{nc} = \mathbf{K}\mathbf{R}_n\mathbf{K}^{\mathrm{T}} , \tag{17}
$$

and the signal after compression is

$$
\mathbf{s}_c = \sqrt{\frac{G_a}{L}}\mathbf{K}\left(\begin{bmatrix}\mathbf{s}^{\mathrm{T}} & \mathbf{s}^{\mathrm{T}} & \cdots & \mathbf{s}^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}\right). \tag{18}
$$

V. Performance Analysis

We wish to evaluate the multibranch receiver architecture shown in Figure 1 with respect to detection of a known signal. Previous work [3-5] has shown that detection performance of a compressive receiver is degraded due to the SNR loss inherent in compressing a signal with N degrees of freedom into an M-dimensional measurement space. Furthermore, the measurement matrix structure of a single-branch receiver (see Figure 2) is restrictive. A multibranch architecture adds additional flexibility in designing and implementing a compressive receiver, but the quality of the LNA and power dividers will impact detection performance. The purpose of the LNA is to dominate the noise figure of the receiver such that additional components do not further degrade SNR. But if the input signal is wideband, which may be driving the compressive implementation in the first place, it could be difficult to find a cost-effective, high-quality LNA.

We evaluate detection performance for a sinusoidal signal that has been compressed with a Gaussian random measurement matrix. Once realizations of the sensing matrices have been generated, they are assumed to be known. We assume that any frequency within the bandwidth B is possible and that the receiver must be capable, on average, of detecting any frequency. This assumption prevents the receiver from using a measurement kernel that is matched to a particular frequency, in which case the receiver could be perfectly compressive by collecting a single sample at the output of a matched filter. In other words, the measurement kernels must be incoherent with all signals in the allowable bandwidth, rather than being fully matched to a particular frequency. We assume that after compression, detection is performed by implementing a bank of filters matched to different frequencies. For any given bin, the detection problem reduces to detection of a known signal in Gaussian noise.

We performed Monte Carlo simulation to obtain our results. The maximum allowable sinusoidal frequency was 100 MHz, and the input signal duration was 0.5 us. Nyquist-rate sampling at 200 MHz would result in 100 samples. We evaluated three cases of the compressive receiver: a single-branch receiver sampling at 20 MHz, a two-branch receiver sampling at 10 MHz, and a four-branch receiver sampling at 5 MHz. Therefore, each compressive receiver operates at a compression ratio of 10. The input noise power was normalized, such that the definition of SNR used was

$$SNR = \alpha^2 / P_i = \alpha^2 . \qquad (19)$$

where α is the signal amplitude. The Monte Carlo trials included independent realizations of the measurement kernels.

Figure 3 shows probability of miss (P_m) when $G_a = F_a = 1$, which is the case where the LNA is not present. We observe approximately 10 dB loss in performance between Nyquist-sampling and the single-branch receiver, which is expected for a compression ratio of 10. However, because the LNA is absent, the multibranch receivers suffer additional loss. The noise figure of an ideal power divider is equal to the number of divisions, and the multibranch receivers show an additional loss approximately equal to L, the number of branches.

Figure 4 shows the opposite extreme where the LNA has 20 dB gain and an ideal noise figure of $F_a = 1$. In this case, the LNA dominates the noise figure of the receiver, regardless of the number of branches, and the various configurations perform nearly the same. Figure 5 shows performance for an LNA with 20 dB gain and varying noise figure.

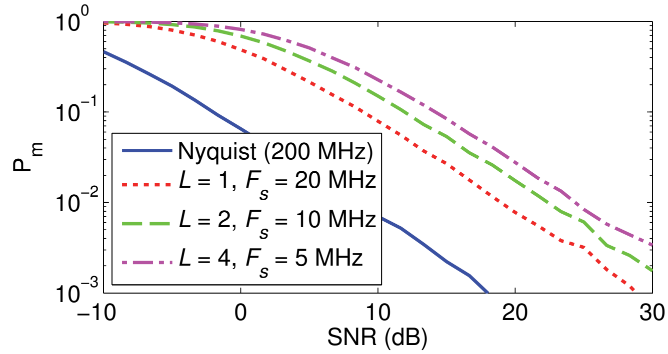

Figure 3. Probability of miss when $G_a = F_a = 1$ (no LNA present).

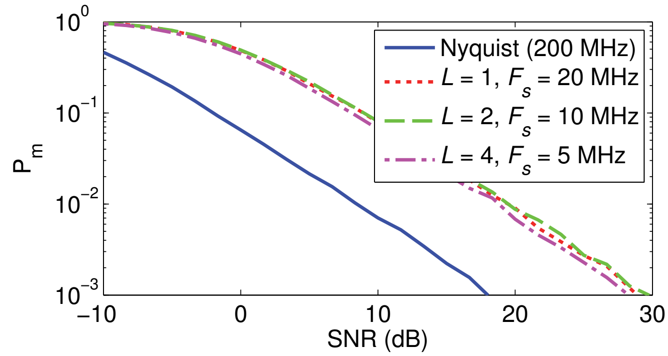

Figure 4. Probability of miss when $G_a = 100$, $F_a = 1$ (Ideal LNA).

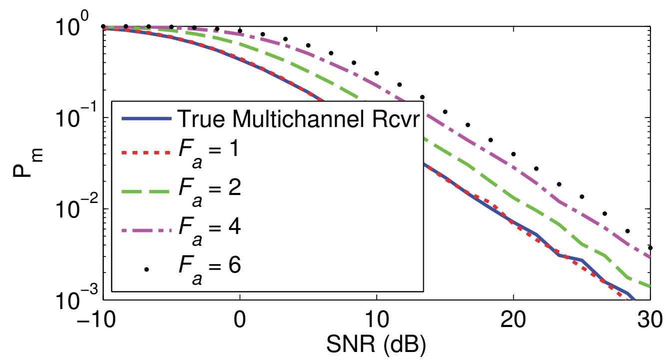

Figure 5. Probability of miss for $G_a = 100$ and varying F_a. Also a comparison to multichannel receiver that doesn't require a power divider. $L = 4$ and a sample rate of 5 MHz per branch.

VI. Conclusions

Most of the CS literature deals with reconstruction of sparse signals from undersampled, but high-SNR, data. Radar systems often operate in the low-SNR regime; therefore, it is useful to consider the impacts of compressive sampling on radar detection performance. We have evaluated the detection performance of various compressive radar receiver configurations comprising different numbers of parallel compression branches. Although parallel structures offer additional flexibility in the design of the compression kernels, in the absence of an ideal wideband LNA they will suffer from SNR loss when the signal is split by a power divider. This loss is in addition to the loss that any RF receiver will incur due to the compression of N-dimensional signal and noise into a smaller M-dimensional measurement space.

Acknowledgment

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