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Robust adaptive beamforming based on interference covariance matrix sparse reconstruction

Yujie Gu [a,*], Nathan A. Goodman [a], Shaohua Hong [b], Yu Li [c]

[a] School of Electrical and Computer Engineering, Advanced Radar Research Center, University of Oklahoma, Norman, OK 73019, USA
[b] School of Information Science and Engineering, Xiamen University, Xiamen, China
[c] Department of Electronic Engineering, East China University of Science and Technology, Shanghai, China

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ABSTRACT

Adaptive beamformers are sensitive to model mismatch, especially when the desired signal is present in the training data. In this paper, we reconstruct the interference-plus-noise covariance matrix in a sparse way, instead of searching for an optimal diagonal loading factor for the sample covariance matrix. Using sparsity, the interference covariance matrix can be reconstructed as a weighted sum of the outer products of the interference steering vectors, the coefficients of which can be estimated from a compressive sensing (CS) problem. In contrast to previous works, the proposed CS problem can be effectively solved by use of *a priori* information instead of using l_1-norm relaxation or other approximation algorithms. Simulation results demonstrate that the performance of the proposed adaptive beamformer is almost always equal to the optimal value.

1. Introduction

Adaptive beamforming is used to detect and estimate the signal-of-interest at the output of a sensor array by means of adaptive spatial filtering and interference suppression. It has been widely used in radar, sonar, seismology, radio astronomy, wireless communications, acoustics, medical imaging, and other areas [1,2]. When there is no required knowledge of direction, blind source separation based beamforming tries to recover the source signals relying on the properties of the signals, such as the constant modulus especially in wireless communication [3,4] (see also Chapter 6 of [2] and the references therein). Instead, when the directions of the source signals are available, the Capon adaptive beamformer is an optimal spatial filter that maximizes the array output signal-to-interference-plus-noise ratio (SINR) [1]. However, it is also known to be sensitive to model mismatch, especially when the desired signal is present in the training data. In such a case, the Capon beamformer suffers severe performance degradation. In addition, in practical applications the required interference-plus-noise covariance matrix cannot be perfectly estimated due to the limited training samples. Therefore, adaptive beamforming approaches must be robust against covariance matrix uncertainty.

Diagonal loading is a simple and well-known robust adaptive beamforming technique [5]. However, there is no clear guideline to choose an optimal loading factor in different scenarios. Worst-case performance optimization [6,7] can also be regarded as a diagonal loading technique; however, the worst case does not always occur, and the norm upper-bound of the mismatch vector is usually *a priori* unknown. Hence, worst-case optimization is still suboptimal. In the past years, some user parameter-free adaptive beamforming algorithms were proposed (see, for example, [8], and the references therein). Unfortunately, these techniques obtain estimates of the theoretical covariance matrix of the received signal, instead of the required interference-plus-noise covariance matrix. More recently, covariance matrix reconstruction methods were proposed [9,10]. In [9], the covariance matrix was reconstructed by locating the nulls of the beampattern of the Capon

* Corresponding author. Tel.: +1 405 474 0527.
E-mail addresses: guyujie@ou.edu, guyujie@ou.edu (Y. Gu), goodman@ou.edu (N.A. Goodman), hongsh@xmu.edu.cn (S. Hong), liyu@ecust.edu.cn (Y. Li).

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beamformer. However, all interference powers are set to the largest eigenvalue of the sample covariance matrix, which is not optimal. In addition, the number of sources is also difficult to determine. In [10], the covariance matrix was reconstructed based on the Capon spatial spectrum, which usually underestimates the interference powers. Furthermore, the computational complexity is comparatively large because of the integral operation.

Considering the fact that the number of sources is typically less than the number of sensors in array signal processing, in this paper we reconstruct the interference-plus-noise covariance matrix in a sparse way. The reconstructed interference covariance matrix is a linear combination of the outer products of the interference steering vectors weighted by their individual powers, which can be estimated from a compressive sensing (CS) problem. This approach allows the desired signal to be removed out from the covariance matrix reconstruction; hence, there will be no signal component in the reconstructed covariance matrix, which mitigates the signal self-nulling problem. In the last decade, many signal recovery algorithms were proposed in the field of CS, such as l_1-norm convex relaxation [11,12] and greedy iterative algorithm [13] (see also [14] and the references therein). Unlike the previous works which mainly exploited the sparsity or compressibility, the proposed CS problem in this paper can be effectively solved by use of *a priori* information of the directions of the source signals, which can be estimated in advance. And hence, a closed-form solution of the CS problem can be derived. Numerical examples demonstrate that the performance of the proposed adaptive beamforming algorithm is nearly equal to the optimal value over a wide range of signal-to-noise ratios (SNRs). Meanwhile, the technique has low computational complexity.

2. The signal model

The output of a narrowband adaptive beamformer with M omni-directional sensors at time k is given by

$$y(k) = \mathbf{w}^H \mathbf{x}(k), \tag{1}$$

where $\mathbf{w} = [w_1, \ldots, w_M]^T \in \mathcal{C}^M$ is the beamformer weight vector, and $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and Hermitian transpose, respectively. The array received vector $\mathbf{x}(k) = [x_1(k), \ldots, x_M(k)]^T \in \mathcal{C}^M$ can be represented as

$$\mathbf{x}(k) = \mathbf{x}_s(k) + \mathbf{x}_i(k) + \mathbf{x}_n(k), \tag{2}$$

where $\mathbf{x}_s(k) = \mathbf{a}s(k)$, $\mathbf{x}_i(k)$, and $\mathbf{x}_n(k)$ are statistically independent components of the desired signal, interference, and noise, respectively. In the desired signal term, $\mathbf{a} \in \mathcal{C}^M$ is the spatial steering vector of the signal waveform $s(k)$.

The optimal weight vector $\mathbf{w}$ can be obtained by maximizing the beamformer output SINR as

$$\mathrm{SINR} \triangleq \frac{E\{|\mathbf{w}^H \mathbf{x}_s|^2\}}{E\{|\mathbf{w}^H(\mathbf{x}_i + \mathbf{x}_n)|^2\}} = \frac{\sigma_s^2 |\mathbf{w}^H \mathbf{a}|^2}{\mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w}}, \tag{3}$$

where $\sigma_s^2 \triangleq E\{|s(k)|^2\}$ is the signal power, $\mathbf{R}_{i+n} \triangleq E\{(\mathbf{x}_i(k) + \mathbf{x}_n(k))(\mathbf{x}_i(k) + \mathbf{x}_n(k))^H\} \in \mathcal{C}^{M \times M}$ is the interference-plus-noise covariance matrix, and $E\{\cdot\}$ denotes statistical expectation. The SINR maximization problem (3) is mathematically equivalent to the minimum variance distortionless response (MVDR) problem [15]:

$$\min_{\mathbf{w}} \quad \mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w} \quad \text{subject to} \quad \mathbf{w}^H \mathbf{a} = 1, \tag{4}$$

which solution

$$\mathbf{w}_{opt} = \frac{\mathbf{R}_{i+n}^{-1} \mathbf{a}}{\mathbf{a}^H \mathbf{R}_{i+n}^{-1} \mathbf{a}}, \tag{5}$$

is sometimes referred to as the Capon beamformer. From this principle of MVDR, several robust adaptive beamforming algorithms have been developed and successfully applied in a wide range of areas (see [16] and the references therein).

Since the exact interference-plus-noise covariance matrix $\mathbf{R}_{i+n}$ is not easy available even in signal-free applications, it is usually substituted by the sample covariance matrix $\hat{\mathbf{R}} = 1/K \sum_{k=1}^{K} \mathbf{x}(k)\mathbf{x}^H(k)$ with K training snapshots, and the obtained adaptive beamformer $\mathbf{w}_{SMI} = \hat{\mathbf{R}}^{-1}\mathbf{a}/\mathbf{a}^H \hat{\mathbf{R}}^{-1}\mathbf{a}$ is called the sample matrix inversion (SMI) adaptive beamformer [17]. Whenever there is a desired signal, the SMI beamformer is in essence the minimum power distortionless response (MPDR) beamformer [1] instead of the MVDR beamformer (5). As K increases, $\hat{\mathbf{R}}$ will converge to its theoretical version $\mathbf{R} = \sigma_s^2 \mathbf{a}\mathbf{a}^H + \mathbf{R}_{i+n}$, and the corresponding SINR will approach the optimal value as $K \to \infty$ under stationary and ergodic assumptions. However, when the number of snapshots K is small, the large gap between $\hat{\mathbf{R}}$ and $\mathbf{R}$ is known to dramatically affect the performance of the SMI beamformer, especially when there is a desired signal in the training samples [5,18].

In previous works, researchers have focused on finding the optimal loading factor for $\hat{\mathbf{R}}$, which inevitably results in performance degradation, especially at high SNRs (see [8] and the references therein). The main reason is that the signal is always active in any kind of diagonal loading beamformers, and its effect becomes more and more pronounced with the increase of SNR [10]. In order to avoid the self-nulling phenomenon, in this paper, we will reconstruct the desired interference-plus-noise covariance matrix $\mathbf{R}_{i+n}$ directly, rather than searching for the potential optimal diagonal loading factor.

3. The proposed algorithm

In order to reconstruct the interference-plus-noise covariance matrix $\mathbf{R}_{i+n}$, we need to know the steering vectors of all interferences and their powers, together with the noise power. When the number of interferences, their locations, and their powers are unknown, the covariance matrix $\mathbf{R}_{i+n}$ can be estimated as [10]

$$\hat{\mathbf{R}}_{i+n} = \int_{\overline{\Theta}} \hat{p}_{Capon}(\theta) \mathbf{d}(\theta) \mathbf{d}^H(\theta) \, d\theta \tag{6}$$

where $\mathbf{d}(\theta)$ is the steering vector associated with a hypothetical direction θ based on the known array structure.

$$\hat{p}_{Capon}(\theta) = \frac{1}{\mathbf{d}^H(\theta) \hat{\mathbf{R}}^{-1} \mathbf{d}(\theta)} \tag{7}$$

is the Capon spatial power spectrum estimator [15], and $\overline{\Theta}$ is the complement sector of Θ. That is to say, $\overline{\Theta} \cap \Theta = \emptyset$ and

$\overline{\Theta} \cup \Theta$ covers the whole spatial domain. In general, the desired signal is assumed to be located in a known angular sector Θ, which is distinguishable from the locations of the interferences. And hence, the covariance matrix estimator $\hat{\mathbf{R}}_{i+n}$ collects all interference and noise in the out-of-sector $\overline{\Theta}$.

In array signal processing, the number of sources is typically assumed to be less than the number of sensors. That is to say, the sources are of sparse nature in the observation field. In such case, the integral operation of (6) over the entire $\overline{\Theta}$ is unnecessary. Instead, the interference-plus-noise covariance matrix $\mathbf{R}_{i+n}$ can be reconstructed based on this sparsity of sources. As we know, l_0-norm (denoted by $\|\cdot\|_0$) equals the count of nonzero entries of a vector, and is an ideal measure of sparsity. Therefore, a sparsity-constrained optimization problem for determining the source locations and their powers can be formulated as

$$\min_{\mathbf{P},\sigma_n^2} \quad \|\hat{\mathbf{R}} - \mathbf{D}\mathbf{P}\mathbf{D}^H - \sigma_n^2\mathbf{I}\|_F^2 + \gamma\|\mathbf{p}\|_0$$
$$\text{subject to} \quad \mathbf{p} \succcurlyeq \mathbf{0}, \quad \sigma_n^2 > 0, \tag{8}$$

where $\mathbf{p} \in \mathcal{R}_+^N$ is the spatial spectrum distribution on the sample grid of all locations of interest $\{\theta_1, \theta_2, \ldots, \theta_N\}$, $\mathbf{P} = diag\{\mathbf{p}\}$ is the corresponding diagonal matrix, $\mathbf{D} = [\mathbf{d}(\theta_1), \mathbf{d}(\theta_2), \ldots, \mathbf{d}(\theta_N)] \in \mathcal{C}^{M\times N}$ is the array manifold matrix, σ_n^2 is the noise power, $\mathbf{I}$ is an identity matrix, and $\|\cdot\|_F$ denotes the Frobenius norm of a matrix. The number of potential sources N will typically be much greater than the number of sources L or the number of sensors M. The optimization problem (8) is a CS problem [14]. The idea behind (8) is intuitive in the sense that it tries to find the sparsest spatial spectrum $\mathbf{p}$ and the noise power σ_n^2 such that the estimated theoretical covariance matrix $\mathbf{D}\mathbf{P}\mathbf{D}^H + \sigma_n^2\mathbf{I}$ approximates $\hat{\mathbf{R}}$, where the parameter γ controls the tradeoff between the sparsity of the spectrum and the residual norm.

However, it is well known that (8) is a difficult combinatorial optimization problem due to the l_0-norm, and is intractable for even moderately sized problems. In the past years, many approximations have been devised, such as greedy approximations [13,19] and $l_p(p \le 1)$ relaxations [14,20]. When the solution $\mathbf{p}$ is sufficiently sparse, the l_0-norm can be approximately replaced by the l_1-norm, which leads to the basis pursuit denoising (BPDN) problem [11] as

$$\min_{\mathbf{P},\sigma_n^2} \quad \|\hat{\mathbf{R}} - \mathbf{D}\mathbf{P}\mathbf{D}^H - \sigma_n^2\mathbf{I}\|_F^2 + \gamma\|\mathbf{p}\|_1$$
$$\text{subject to} \quad \mathbf{p} \succcurlyeq \mathbf{0}, \quad \sigma_n^2 > 0. \tag{9}$$

Unlike (8), (9) is convex and can be solved using convex optimization software. Besides the BPDN, the LASSO (least absolute shrinkage and selection operator) [12] is another popular formulation based on l_1-norm relaxation. However, the solution is not absolutely sparse because of the l_1-norm relaxation. In addition, the regularization parameter γ is also difficult to determine in different scenarios. Either overestimation or underestimation will sacrifice the balance between data-fidelity and sparsity, which leads to performance degradation of the system.

Unlike previous works that focus on l_1-norm relaxation, in this paper the CS problem (8) will be solved by decomposing it into two separate sub-problems: first, we find the direction-of-arrival (DOA) support of the sources by exploiting the available training data; second, we estimate the powers of these sources via an inequality-constrained least squares problem operating on the DOAs found in the first step. The combination of these two steps represents an approximation to the solution to (8).

In array signal processing, the DOAs are usually estimated either from a spectral search algorithm or a *search-free* method (via polynomial rooting) (see [1,21] and the references therein). For the sake of explanation, in this paper, we simply use the classical Capon spatial spectrum $\hat{\mathbf{p}}_{Capon}$ (7) to estimate the DOAs of sources, namely the support of the sparse vector defined in the l_0-norm minimization problem of (8), although there are many sophisticated DOA estimation algorithms for uncorrelated and/or correlated sources (see, for example, [1,20] and the references therein). Let $\boldsymbol{\theta}_p$ denote the set of directions corresponding to the peaks of $\hat{\mathbf{p}}_{Capon}$ on the entire angular sector, for which the cardinality (denoted by $|\cdot|$ of a set) is usually greater than the number of sources because of the spurious peaks, i.e., $|\boldsymbol{\theta}_p| = \|\mathbf{p}\|_0 > L$. In order to $\min_{\mathbf{p}}\|\mathbf{p}\|_0$, a common idea is to remove the spurious peaks by setting a threshold, such as the noise power $\hat{\sigma}_n^2$, which can be approximately estimated as the minimum eigenvalue of $\hat{\mathbf{R}}$ [22]. In theory, there are $M-L$ same eigenvalues as the actual noise power σ_n^2. However, when the number of snapshots is limited, the minimum eigenvalue of $\hat{\mathbf{R}}$ is always less than the noise power σ_n^2. And hence, if a peak's value is less than the threshold, it can be regarded as a spurious peak and removed from $\boldsymbol{\theta}_p$. After removing the spurious peaks, the residual set can be denoted as $\tilde{\boldsymbol{\theta}}_p = \{\tilde{\theta}_{p,1} \ldots, \tilde{\theta}_{p,Q}\}$, which is of cardinality $|\tilde{\boldsymbol{\theta}}_p| = Q < M$. And hence, $\min_{\mathbf{p}}\|\mathbf{p}\|_0 = Q$.

Because of the high resolution ability, the DOA of the desired signal can be located by searching for the peak of $\hat{\mathbf{p}}_{Capon}$ in Θ, and the corresponding steering vector can be denoted as $\tilde{\mathbf{a}}$. When there is no peak in Θ, which is very common at low SNRs, we must assume a DOA θ_s for the desired signal using some prior knowledge, although the assumed direction may be different than the actual DOA, and $\tilde{\mathbf{a}} = \mathbf{d}(\theta_s)$.

After finding the DOA support, the CS problem (8) degenerates into an inequality-constrained least squares problem:

$$\min_{\mathbf{p}(\tilde{\boldsymbol{\theta}}_p)} \quad \|\hat{\mathbf{R}} - \hat{\sigma}_n^2\mathbf{I} - \mathbf{D}(\tilde{\boldsymbol{\theta}}_p)\mathbf{P}(\tilde{\boldsymbol{\theta}}_p)\mathbf{D}^H(\tilde{\boldsymbol{\theta}}_p)\|_F^2$$
$$\text{subject to} \quad \mathbf{p}(\tilde{\boldsymbol{\theta}}_p) \succ \mathbf{0}, \tag{10}$$

where $\mathbf{p}(\tilde{\boldsymbol{\theta}}_p) \in \mathcal{R}_{++}^Q$ is the power distribution on $\tilde{\boldsymbol{\theta}}_p$, $\mathbf{P}(\tilde{\boldsymbol{\theta}}_p) = diag\,\{\mathbf{p}(\tilde{\boldsymbol{\theta}}_p)\}$ is the corresponding diagonal matrix, and $\mathbf{D}(\tilde{\boldsymbol{\theta}}_p) = [\mathbf{d}(\tilde{\theta}_{p,1}), \ldots, \mathbf{d}(\tilde{\theta}_{p,Q})] \in \mathcal{C}^{M\times Q}$ is the corresponding array manifold matrix. Because the performance of adaptive beamformer is not very sensitive to errors in the noise power, $\hat{\sigma}_n^2$ is taken to be the minimum eigenvalue of $\hat{\mathbf{R}}$, instead of an optimization variable as seen in (8).

Without the inequality constraint, the closed-form solution to (10) can be given by

$$\mathbf{p}(\tilde{\boldsymbol{\theta}}_p) = (\mathbf{G}^H\mathbf{G})^{-1}\mathbf{G}^H\mathbf{r} \tag{11}$$

where $\mathbf{G} \equiv [\mathrm{vec}(\mathbf{d}(\tilde{\theta}_{p,1})\mathbf{d}^H(\tilde{\theta}_{p,1})), \ldots, \mathrm{vec}(\mathbf{d}(\tilde{\theta}_{p,Q})\mathbf{d}^H(\tilde{\theta}_{p,Q}))] \in \mathcal{C}^{M^2 \times Q}$ and $\mathbf{r} = \mathrm{vec}(\hat{\mathbf{R}} - \hat{\sigma}_n^2 \mathbf{I}) \in \mathcal{C}^{M^2}$ are obtained by stacking the array responses and the sample covariance matrix subtracted by a noise covariance matrix, respectively. Then, the estimated spatial spectrum of (8) is Q-sparse as

$$\mathbf{p}(\theta) = \begin{cases} \mathbf{p}(\tilde{\boldsymbol{\theta}}_p), & \theta \in \tilde{\boldsymbol{\theta}}_p \\ 0, & \theta \notin \tilde{\boldsymbol{\theta}}_p \end{cases}. \tag{12}$$

Namely, only Q entries of $\mathbf{p}(\theta)$ are nonzero and all other $(N-Q)$ are zero. We illustrate the proposed sparse spatial spectrum in Fig. 1, where the source signal steering vectors are exactly known as the first example in Section 4.

However, when the source powers are small, there may be one or more negative entries in $\mathbf{p}(\tilde{\boldsymbol{\theta}}_p)$ as given in (11). We can assume without loss of generality that the q-th entry of $\mathbf{p}(\tilde{\boldsymbol{\theta}}_p)$ is negative, $p(\tilde{\theta}_{p,q}) < 0$. In this case, the inequality constraint of (10) will not be satisfied, and the solution (11) should be modified. A simple method is to force $p(\tilde{\theta}_{p,q})$ to be a small positive number $\delta > 0$ (for example, $\delta = 10^{-5}$ will be used in our simulations), and the power estimation of the remaining sources will be modified as

$$\overline{\mathbf{p}}(\overline{\boldsymbol{\theta}}_p) = (\overline{\mathbf{G}}^H \overline{\mathbf{G}})^{-1} \overline{\mathbf{G}}^H \overline{\mathbf{r}} \tag{13}$$

where $\overline{\boldsymbol{\theta}}_p = [\tilde{\theta}_{p,1}, \ldots, \tilde{\theta}_{p,q-1}, \tilde{\theta}_{p,q+1}, \ldots, \tilde{\theta}_{p,Q}]^T \in \mathcal{R}^{(Q-1)}$, $\overline{\mathbf{G}} \equiv [\mathrm{vec}(\mathbf{d}(\tilde{\theta}_{p,1})\mathbf{d}^H(\tilde{\theta}_{p,1})), \ldots, \mathrm{vec}(\mathbf{d}(\tilde{\theta}_{p,q-1})\mathbf{d}^H(\tilde{\theta}_{p,q-1})), \mathrm{vec}(\mathbf{d}(\tilde{\theta}_{p,q+1})\mathbf{d}^H(\tilde{\theta}_{p,q+1})), \ldots, \mathrm{vec}(\mathbf{d}(\tilde{\theta}_{p,Q})\mathbf{d}^H(\tilde{\theta}_{p,Q}))] \in \mathcal{C}^{M^2 \times (Q-1)}$, and $\overline{\mathbf{r}} \equiv \mathrm{vec}(\hat{\mathbf{R}} - \hat{\sigma}_n^2 \mathbf{I} - \delta \mathbf{d}(\tilde{\theta}_{p,q})\mathbf{d}^H(\tilde{\theta}_{p,q})) \in \mathcal{C}^{M^2}$. In other words, we re-calculate the source powers after fixing the power of small sources, which results in a modified spatial spectrum as

$$\mathbf{p}(\theta) = \begin{cases} \overline{\mathbf{p}}(\overline{\boldsymbol{\theta}}_p), & \theta \in \overline{\boldsymbol{\theta}}_p \\ \delta, & \theta = \tilde{\theta}_{p,q} \\ 0, & \theta \notin \tilde{\boldsymbol{\theta}}_p \end{cases}, \tag{14}$$

which is still Q-sparse.

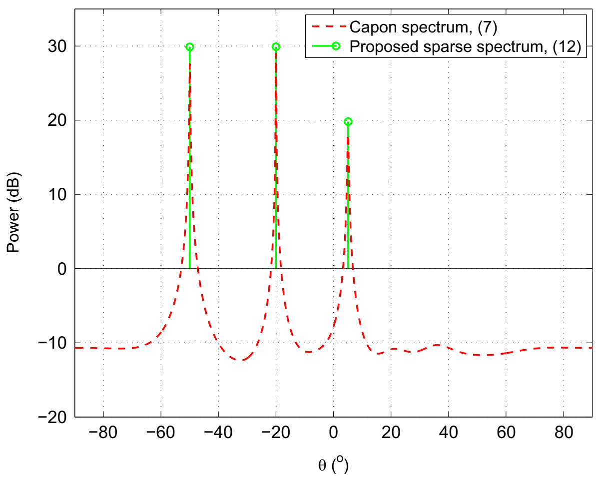

Fig. 1. Spatial spectrum comparison from one trial, where three sources from DOAs of $-50°$, $-20°$ and $5°$ are with SNR = 30 dB, 30 dB and 20 dB, respectively.

Using the Q-sparse spatial spectrum $\mathbf{p}(\theta)$, the interference-plus-noise covariance matrix can be reconstructed as

$$\tilde{\mathbf{R}}_{i+n} = \sum_{j=1, j \neq q}^{Q} p(\tilde{\theta}_{p,j}) \mathbf{d}(\tilde{\theta}_{p,j}) \mathbf{d}^H(\tilde{\theta}_{p,j}) + \hat{\sigma}_n^2 \mathbf{I}, \tag{15}$$

where $\mathbf{d}(\tilde{\theta}_{p,j})\mathbf{d}^H(\tilde{\theta}_{p,j})$ is the outer product of the j-th interference steering vector $\mathbf{d}(\tilde{\theta}_{p,j})$. By use of the sparse characteristics, the integral operation of (6) can be effectively simplified to a summation operation (15).

Substituting $\tilde{\mathbf{R}}_{i+n}$ and $\tilde{\mathbf{a}}$ into (5), we can get the adaptive beamformer based on interference covariance matrix sparse reconstruction as

$$\mathbf{w}_S = \frac{\tilde{\mathbf{R}}_{i+n}^{-1} \tilde{\mathbf{a}}}{\tilde{\mathbf{a}}^H \tilde{\mathbf{R}}_{i+n}^{-1} \tilde{\mathbf{a}}}. \tag{16}$$

The interference covariance matrix sparse reconstruction-based adaptive beamforming algorithm is summarized in Table 1.

The computational complexity of the proposed algorithm is $\mathcal{O}(NM^2)$ with $N \gg M$, which is mainly dominated by the spectral search. If a *search-free* DOA estimation technique [21] is adopted, the computational complexity can be further decreased to $\mathcal{O}(\max(M^3, Q^2M^2))$, where $\mathcal{O}(M^3)$ is the complexity of *search-free* DOA estimation and $\mathcal{O}(Q^2M^2)$ is the complexity of power estimation (11). Therefore the proposed beamforming algorithm has complexity slightly larger than the DOA estimation algorithm. Meanwhile, the computational complexities of covariance matrix reconstruction methods [9] and [10] are $\mathcal{O}(\max((|\overline{\Theta}|/|\overline{\Theta} \cup \Theta|)NM, M^3))$ and $\mathcal{O}((|\overline{\Theta}|/|\overline{\Theta} \cup \Theta|)NM^2)$, respectively. Note however that if the spatial estimate of the sources in the whole region is desired, the SMI beamformer has complexity $\mathcal{O}(NM^2)$ as well.

4. Simulation results

In our simulations, a uniform linear array (ULA) with $M = 10$ omni-directional sensors spaced half wavelength apart is considered. It is assumed that there is one desired signal from the presumed direction $\theta_s = 5°$ and two uncorrelated interferences from $-50°$ and $-20°$. The interference-to-noise ratio (INR) in each sensor is equal to 30 dB. The additive noise is modeled as a complex circularly symmetric Gaussian zero-mean spatially and temporally white process. When comparing the performance of the adaptive beamforming algorithms in terms of the input SNR, the number of snapshots is fixed to be $K = 30$. In the performance comparison of mean output SINR versus the number of samples, the SNR in each sensor is set to be fixed at 20 dB. For each scenario, 1000 Monte-Carlo trials are performed.

Table 1
Interference covariance matrix sparse reconstruction-based adaptive beamforming algorithm.

Step 1: Estimate the DOAs of the sources.
Step 2: Solve (10) to get the Q-sparse spatial spectrum estimator $\mathbf{p}(\theta)$ (12) or (14).
Step 3: Reconstruct the interference-plus-noise covariance matrix $\tilde{\mathbf{R}}_{i+n}$ (15).
Step 4: Calculate the adaptive beamformer $\mathbf{w}_S$ (16) with the modified steering vector $\tilde{\mathbf{a}}$.

The proposed adaptive beamformer (16) is compared to two other different covariance matrix reconstruction-based adaptive beamformers [9] and [10], and the adaptive beamformer based on covariance matrix reconstruction plus steering vector estimation [10], because most of the existing adaptive beamformers will suffer performance degradation especially at high SNRs [10]. As a reference, the worst-case-based beamformer with parameter $\varepsilon = 0.3$ M of [6] is also shown in our simulations. Meanwhile, the nominal steering vector is normalized so that $\|\mathbf{a}\|^2 = \mathbf{a}^H\mathbf{a} = M(=10)$ [6,7]. As recommended in [10], Θ is set to be $[\theta_s - 5^\circ, \theta_s + 5^\circ]$ (namely $[0^\circ, 10^\circ]$), and the corresponding out-of-sector is $\overline{\Theta} = [-90^\circ, \theta_s - 5^\circ) \cup (\theta_s + 5^\circ, 90^\circ]$ (namely $[-90^\circ, 0^\circ) \cup (10^\circ, 90^\circ]$). The sampling grid is uniform in $\Theta \cup \overline{\Theta}$ with 0.1° increment between adjacent grid points. Performance is presented in terms of deviation from the optimal SINR.

4.1. Example 1: exactly known signal steering vector

In the first example, we consider an ideal case when the spatial signatures both of the signal and of the interferences are exactly known. Note that even in this ideal case, the presence of the desired signal in the training data may substantially degrade the output performance of adaptive beamformers as compared with the signal-free training data case [1,6,10]. However, it can be seen from Fig. 2(a) that the performance of the proposed beamformer is almost always equal to the optimal SINR for all values of SNR from -30 to 50 dB, which illustrates its high dynamic range. Specifically,

$$\text{SINR} = \|\mathbf{a}\|^2 \text{SNR} = M \times \text{SNR}, \tag{17}$$

which achieves the design goal of the adaptive beamformers. Furthermore, the proposed beamformer outperforms the others. In detail, there is about 0.15 dB performance losses for the adaptive beamformer [10], which is because the Capon spatial spectrum (7) underestimates the interference powers. In addition, there is about 0.5 dB performance losses for the adaptive beamformer [9] at high SNRs, which is because in such cases the largest eigenvalue of the sample covariance matrix adopted in [9] is corresponding to the desired signal instead of the interferences. It should be noted that the signal power is 100 times larger than the interference power in the case of SNR=50 dB, which can be used to illustrate the situation when the signal-to-interference ratio (SIR) approximately approaches to ∞. Fig. 2(b) shows the performance of the methods tested versus the number of training snapshots K, where the performance curve of the worst-case-based beamformer is not plotted because it is much worse than the others at high SNRs [10].

4.2. Example 2: random sources look direction mismatch

In the second example, a more practical scenario with random DOA mismatch is considered. The random DOA mismatch of both the desired signal and the interferences are uniformly distributed in $[-4^\circ, 4^\circ]$. That is to say, the actual DOA of signal is uniformly distributed in $[1^\circ, 9^\circ]$, and the DOAs of interferences are uniformly distributed in $[-54^\circ, -46^\circ]$ and $[-24^\circ, -16^\circ]$. Note that the random DOAs of the signal and the interferences change from trial to trial but remain fixed from snapshot to snapshot.

It can be seen from Fig. 3(a) that the performance of the proposed beamformer is much closer to the optimal SINR than others. There is approximately 0.6 dB performance degradation when SNR is less than -10 dB, which is because there may be no peak in the angular sector Θ for the Capon spectrum or the peak's value is less than the threshold; therefore, the presumed DOA is taken as the center of the desired signal sector and the source DOA mismatch will be present. In addition, due to the grid limit, the performance of the proposed adaptive beamformer does not exactly converge to the optimal one when SNR is larger than 0 dB. In detail, the maximum estimation error of source DOAs should be 0.05°, half grid increment of 0.1°, which will degrade the performance of the proposed adaptive beamformer (16) because both the reconstructed covariance matrix $\tilde{\mathbf{R}}_{i+n}$ and the modified steering vector $\tilde{\mathbf{a}}$ depend on the DOA estimation. A possible solution to mitigate the effects of grid limiting is the grid refinement method [20], with which method the maximum estimation

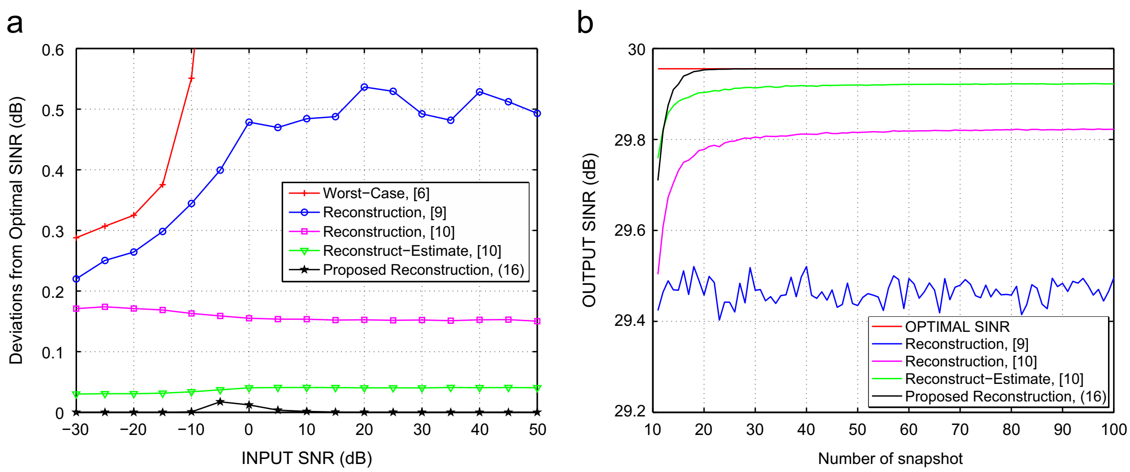

Fig. 2. First example: Exactly known signal steering vector. (a) Deviations from optimal SINR versus SNR, (b) output SINR versus number of snapshots.

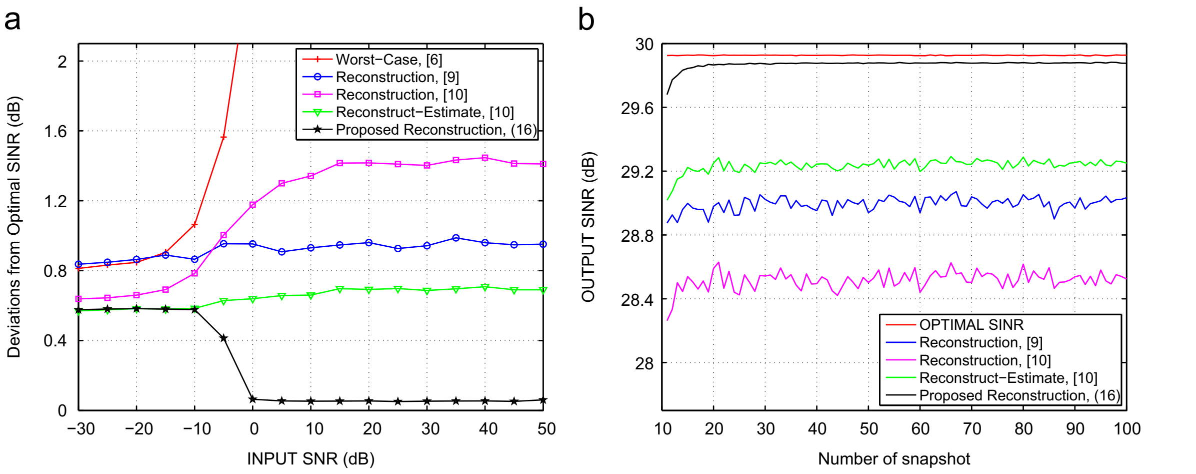

Fig. 3. Second example: Random sources look direction mismatch. (a) Deviations from optimal SINR versus SNR, (b) output SINR versus number of snapshots.

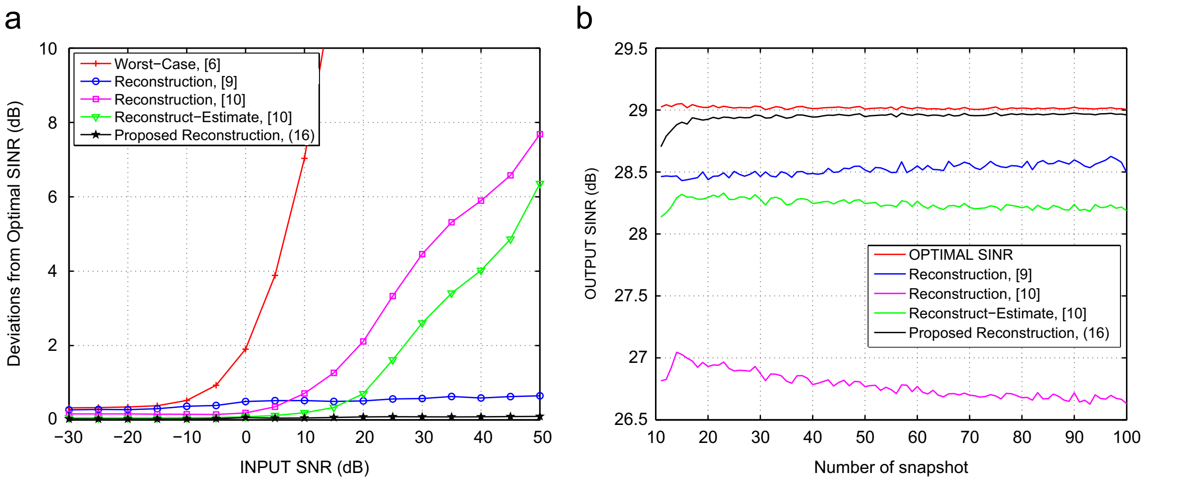

Fig. 4. Third example: Incoherent local scattering. (a) Deviations from optimal SINR versus SNR, (b) output SINR versus number of snapshots.

error of source DOAs can be effectively reduced. And then, the proposed adaptive beamformer $\mathbf{w}_S$ (16) is more close to the optimal beamformer $\mathbf{w}_{opt}$ (5). In addition to the faster convergence rate of the adaptive beamforming based on interference covariance matrix reconstruction [10], the performance of the proposed beamforming algorithm based on interference covariance matrix sparse reconstruction keeps stable with the increase of SNR while others not as shown in Fig. 3(b).

4.3. Example 3: incoherent local scattering

In the third example, we assume incoherent local scattering of the desired signal, which is common in array signal processing due to the multipath scattering effects caused by the presence of local scatters. The signal is assumed to have a time-varying spatial signature as [6,10]

$$\mathbf{a}(k) = s_0(k)\mathbf{d}(\theta_s) + \sum_{l=1}^{4} s_l(k)\mathbf{d}(\theta_l), \tag{18}$$

where $s_l(k) \sim \mathcal{N}(0,1)$, $l = 0,1,2,3,4$ are independently and identically distributed (i.i.d.) zero-mean complex Gaussian random variables changing from snapshot to snapshot, $\theta_l \sim \mathcal{N}(\theta_s, 4^\circ)$, $l = 1,2,3,4$ are the random DOAs changing from run to run while remaining fixed from snapshot to snapshot. This corresponds to the case of incoherent local scattering [23], where the signal covariance matrix $\mathbf{R}_s$ is no longer a rank-one matrix. In the general-rank case, the output SINR should be rewritten as [18]

$$\mathrm{SINR} = \frac{\mathbf{w}^H \mathbf{R}_s \mathbf{w}}{\mathbf{w}^H \mathbf{R}_{i+n} \mathbf{w}}, \tag{19}$$

which is maximized by [18]

$$\mathbf{w}_{\mathrm{opt}} = \mathcal{P}\{\mathbf{R}_{i+n}^{-1}\mathbf{R}_s\}, \tag{20}$$

where $\mathcal{P}\{\cdot\}$ stands for the principal eigenvector of a matrix. It can be seen from Fig. 4 that the proposed beamforming algorithm outperforms than all other methods tested especially at high SNRs. In detail, there is about 7.5 dB performance losses for the adaptive beamformer based on interference covariance matrix reconstruction [10]. The main reason is that the signal-of-interest leaks into the out-of-sector $\overline{\Theta}$ due to the incoherent local scattering, and then the reconstructed covariance matrix $\hat{\mathbf{R}}_{i+n}$ (6) is contaminated by the leaked signal component. However, there is almost no performance losses for the proposed beamformer because it is based on covariance matrix sparse reconstruction with knowledge of the estimated DOAs of sources.

5. Conclusion

In this paper, we proposed a simple, effective adaptive beamforming algorithm, which is robust against covariance matrix uncertainty. When the sources are sparsely distributed, accurate interference covariance matrix reconstruction can be achieved by estimating the sparse spatial spectrum distribution from a CS problem, which provides a quasi-signal-free environment. The proposed CS problem can be effectively solved with *a priori* information (i.e., the estimated source DOAs in array signal processing) rather than l_1-norm relaxation-like approximations. Simulation results demonstrate the effectiveness of the proposed algorithm. Compared with existing techniques, the performance of the proposed method is nearly optimal over a wide range of SNR. In addition, the technique also has low computational complexity.

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