Source Estimation Using Coprime Array: A Sparse Reconstruction Perspective

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***Abstract*— Direction-of-arrival (DOA), power, and achievable degrees-of-freedom (DOFs) are fundamental parameters for source estimation. In this paper, we propose a novel sparse reconstruction-based source estimation algorithm by using a coprime array. Specifically, a difference coarray is derived from a coprime array as the foundation for increasing the number of DOFs, and a virtual uniform linear subarray covariance matrix sparse reconstruction-based optimization problem is formulated for DOA estimation. Meanwhile, a modified sliding window scheme is devised to remove the spurious peaks from the reconstructed sparse spatial spectrum, and the power estimation is enhanced through a least squares problem. Simulation results demonstrate the effectiveness of the proposed algorithm in terms of DOA estimation and power estimation as well as the achievable DOFs.**

***Index Terms*— Coprime array, DOA estimation, power estimation, source enumeration, sparse reconstruction.**

I. Introduction

As a fundamental application in array signal processing, source estimation has been widely used in radar, sonar, acoustics, astronomy, seismology, wireless communications, medical imaging, and other areas (see, for example, [1]–[16], and the references therein). It is common in practice that the number of sources to be estimated is larger than the number of sensors in the array. However, the degrees-of-freedom (DOFs) of the conventional source estimation algorithms are limited by the number of sensors. In general, an array with M physical sensors can identify up to $M - 1$ sources. To detect more sources, additional sensors are required to increase the achievable number of DOFs, which leads to an increased complexity. Therefore, an active research topic has been focused on how to increase the number of DOFs for source estimation.

Although sparse arrays such as minimum redundancy array (MRA) [17] and minimum hole array (MHA) [18], have been proven to enable to increase the number of DOFs, there is no closed form expression for the exact location of sensors in sparse arrays. Until recently, several attractive sparse array configurations have been proposed to support systematical design of sparse arrays. Two important array configurations are the nested array [19] and the coprime array [20]. The former can obtain $\mathcal{O}(M^2)$ DOF with only M physical sensors, the latter can reach up to $\mathcal{O}(MN)$ DOF with $M + N - 1$ physical sensors. Hence, coprime array has attracted great interests in DOFs increase, direction-of-arrival (DOA) estimation, adaptive beamforming, and source localization [21]–[26].

Among these studies, some works have considered how to achieve high-resolution DOA estimation by using the large array aperture of coprime array. For example, a search-free DOA estimation algorithm using the coprime array [22] can achieve near optimal estimation performance in some signal-to-noise ratio (SNR) ranges. By decomposing a coprime array into a pair of sparse uniform linear arrays (ULAs), the DOAs can be estimated by pairing the common peaks in the spatial spectra of the pair of decomposed subarrays [21], [27]. However, the maximum achievable DOFs have not been fully exploited by using coprime array directly.

To take full advantage of the potential DOFs provided by coprime array, a lot of efforts have been conducted on designing effective DOA estimation algorithms using the derived difference coarray. To name a few, a super-resolution spectrum estimation algorithm was proposed in [28] by applying the spatial smoothing technique. In [29]–[31], the idea of sparse recovery was introduced to detect multiple sources. In [32], compressive sensing was introduced to formulate an optimization problem for DOA estimation, where the sparsity of the sources was exploited. More recently, generalized coprime array configurations [33] were developed for DOA estimation, and the Toeplitz property of correlation matrix was also exploited [25]. These algorithms are able to identify more sources than sensors, i.e., the number of DOFs can be increased by using the derived difference coarray. However, there may exist several spurious peaks in the estimated spatial spectrum, which will dramatically affect the overall source estimation performance. In this sense, it still remains a challenging problem to perform accurate DOA estimation with full utilization of potential DOFs provided by coprime array.

DOA, power, and the achievable DOFs are the fundamental parameters for source estimation. Instead of investigating the DOA estimation performance only, in this paper, we simultaneously consider these parameters and develop an effective

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source estimation algorithm from a sparse reconstruction perspective by using a coprime array. Specifically, we formulate an optimization problem for source estimation by minimizing the difference between the derived spatially smoothed covariance matrix and the sparsely reconstructed covariance matrix corresponding to the virtual uniform linear subarray. Considering that there is usually no priori information about the number of the sources, it is difficult to distinguish and remove the spurious peaks in the estimated spatial spectrum. To address this issue, we devise a modified sliding window scheme for source enumeration, which determines the number of sources and removes the spurious peaks. Moreover, the power estimation is also enhanced by solving a least squares problem with the estimated DOAs as the support information. Simulation results demonstrate the superiority of the proposed source estimation algorithm on spectrum characteristic, source enumeration, DOA estimation, and power estimation.

The main contributions of this paper can be categorized as follows.

- We propose a sparse reconstruction-based source estimation algorithm using coprime array, which simultaneously considers the estimation accuracy of DOA and power as well as the number of DOFs;
- We incorporate the spatially smoothed covariance matrix corresponding to the virtual uniform linear subarray to perform DOA estimation from a sparse reconstruction perspective;
- We devise a modified sliding window scheme to remove the spurious peaks from the reconstructed spatial spectrum;
- We utilize the estimated DOAs to enhance the power estimation through solving a least squares problem.

The remainder of this paper is organized as follows. In Section II, we introduce the signal model of the coprime array. In Section III, we elaborate the proposed sparse reconstruction-based source estimation algorithm, and in Section IV, we compare the performance of the proposed algorithm with others. We make our conclusions in Section V.

Throughout this paper, we denote vectors and matrices by lower-case and upper-case bold characters, respectively. The superscripts $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^H$ denote the conjugate, the transpose, and the Hermitian transpose of a vector or a matrix, respectively. $\text{vec}(\cdot)$ denotes the vectorization operator that stacks the column vectors of a matrix one by one. And $\text{diag}(\cdot)$ denotes a diagonal matrix where the vector is in its diagonal or a vector consisting of the diagonal elements of a matrix. The curled inequality symbol $\succeq$ denotes componentwise inequality between vectors. $\|\cdot\|_0$, $\|\cdot\|_1$, and $\|\cdot\|_F$ denote the ℓ_0 norm, ℓ_1 norm, and the Frobenius norm, respectively. $E[\cdot]$ denotes the statistical expectation operator, and $\otimes$ denotes the Kronecker product. Finally, $\mathbf{I}$ denotes the identity matrix with appropriate dimension unless otherwise specified.

II. Signal Model of Coprime Array

Consider a pair of sparse ULAs with M and N sensors, respectively, where M and N are coprime numbers. As shown in Fig. 1(a), the array consisting of M sensors has inter-element spacing of Nd, whereas the array consisting of

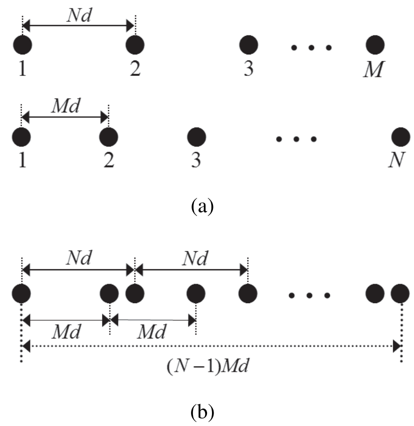


Fig. 1. Coprime array configuration. (a) Coprime pair of sparse ULAs. (b) Aligned coprime array.

N sensors has inter-element spacing of Md. Without loss of generality, we assume that $M < N$, and d is chosen to be $\lambda/2$, where λ denotes the signal wavelength. As Fig. 1(b) shows, a special array geometry called the coprime array can be generated by combining the pair of sparse ULAs [20]. According to the properties of coprime numbers, other than the first sensor serving as the reference, the other sensors do not overlap with each other when the pair of sparse ULAs are aligned. Therefore, the coprime array consists of $M + N - 1$ physical sensors in total, and the last element is located at $(N-1)Md$.

The difference coarray can be calculated as

$$x(m,n) = Mn - Nm, \tag{1}$$

where $0 \le m \le M-1$ and $0 \le n \le N-1$. Since M and N are coprime, MN different values corresponding to the MN combinations of (m,n) can be obtained for $x(m,n)$. Hence, a virtual array with MN nominal sensors locating at $\{(Mn - Nm)d, 0 \le m \le M-1, 0 \le n \le N-1\}$ can be generated by using only $M + N - 1$ sensors, which means the number of DOFs can be increased from $\mathcal{O}(M+N)$ to $\mathcal{O}(MN)$. Moreover, if the negative part of $x(m,n)$ is also included, more than $\mathcal{O}(MN)$ DOFs can be obtained within

$$-M(N-1) \le x(m,n) \le M(N-1). \tag{2}$$

which in most cases is a non-consecutive difference coarray because there exist several missing elements called holes. For example, the elements -11, -8, 8 and 11 are the holes when $M = 3$ and $N = 5$.

To address this problem, we double the aperture of the array with M sensors in Fig. 1(a) to obtain an extended coprime array [28], which consists of $2M + N - 1$ sensors as shown in Fig. 2. Similarly, the difference coarray set derived from the extended coprime array becomes

$$\begin{aligned} S_d = \big\{ \bar{x}(m,n) \,|\, \bar{x}(m,n) = \pm(Mn - Nm), \\ 0 \le m \le 2M-1, 0 \le n \le N-1 \big\}, \end{aligned} \tag{3}$$

from which a consecutive difference coarray subset ranging from $-MN$ to MN can be picked up. Hence, a consecutive

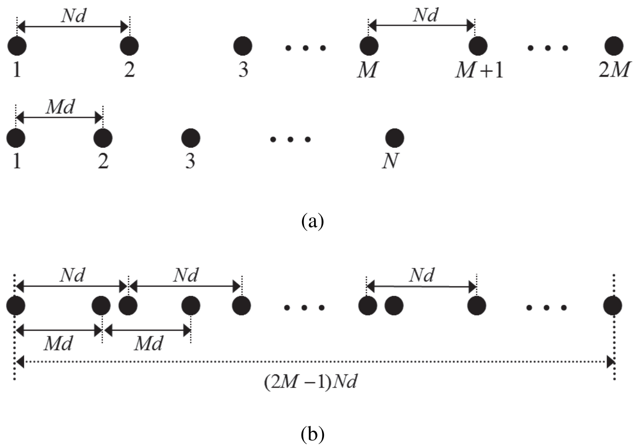


Fig. 2. Extended coprime array configuration. (a) Double the aperture of the array with $M(M < N)$ sensors. (b) Aligned extended coprime array.

virtual ULA with the aperture of $2MNd$ is obtained, which the nominal sensors locate at $\{-MNd, -(MN-1)d, \cdots, -d, 0, d, \cdots, (MN-1)d, MNd\}$.

Assume there are K far-field, uncorrelated, narrowband signals impinging on the extended coprime array from the directions $\boldsymbol{\theta} = [\theta_1, \theta_2, \cdots, \theta_K]^T$. The received signal vector at time index t can be modeled as

$$\mathbf{y}(t) = \sum_{k=1}^{K} \mathbf{a}(\theta_k) s_k(t) + \mathbf{n}(t) = \mathbf{A}(\boldsymbol{\theta})\mathbf{s}(t) + \mathbf{n}(t), \tag{4}$$

where $\mathbf{A}(\boldsymbol{\theta}) = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \cdots, \mathbf{a}(\theta_K)] \in \mathbb{C}^{(2M+N-1)\times K}$ denotes the coprime array steering matrix, $\mathbf{s}(t) = [s_1(t), s_2(t), \cdots, s_K(t)]^T \in \mathbb{C}^K$ denotes the signal waveform vector, and $\mathbf{n}(t) \sim \mathcal{CN}(\mathbf{0}, \sigma_n^2 \mathbf{I})$ denotes the additive Gaussian white noise vector. Here, σ_n^2 denotes the noise power. For simplicity, we use $\mathbf{A}$ to represent the steering matrix $\mathbf{A}(\boldsymbol{\theta})$ throughout the rest of this paper. The k-th column of the steering matrix $\mathbf{A}$,

$$\mathbf{a}(\theta_k) = \left[1, e^{-\jmath \frac{2\pi}{\lambda} u_2 d \sin\theta_k}, \cdots, e^{-\jmath \frac{2\pi}{\lambda} u_{2M+N-1} d \sin\theta_k}\right]^T \tag{5}$$

is the array steering vector corresponding to the k-th source, where $\jmath = \sqrt{-1}$, and $\{u_i d, i = 2, \cdots, 2M+N-1\}$ denotes the sensor positions in the extended coprime array.

The covariance matrix of the received signal vector $\mathbf{y}(t)$ can be expressed as

$$\begin{aligned} \mathbf{R} &= E[\mathbf{y}(t)\mathbf{y}^H(t)] \\ &= \sum_{k=1}^{K} \sigma_k^2 \mathbf{a}(\theta_k)\mathbf{a}^H(\theta_k) + \sigma_n^2 \mathbf{I} \\ &= \mathbf{A}\mathbf{P}\mathbf{A}^H + \sigma_n^2 \mathbf{I}, \end{aligned} \tag{6}$$

where σ_k^2 denotes the signal power of the k-th source, and

$$\mathbf{P} = \operatorname{diag}\left(\left[\sigma_1^2, \sigma_2^2, \cdots, \sigma_K^2\right]^T\right). \tag{7}$$

Considering that $\mathbf{R}$ is unavailable in practice, it is usually replaced by the sample covariance matrix

$$\hat{\mathbf{R}} = \frac{1}{T} \sum_{t=1}^{T} \mathbf{y}(t)\mathbf{y}^H(t), \tag{8}$$

where T denotes the number of snapshots. Note that $\hat{\mathbf{R}}$ is the maximum likelihood estimator of $\mathbf{R}$, and it converges to $\mathbf{R}$ as $T \to \infty$ under stationary and ergodic assumptions [34]. However, when T is small, the large gap between $\hat{\mathbf{R}}$ and $\mathbf{R}$ will affect the estimation performance.

III. Proposed Source Estimation Algorithm Based on Sparse Reconstruction

In this section, we propose a novel sparse reconstruction-based source estimation algorithm using coprime array. First, a full rank covariance matrix corresponding to the virtual uniform linear subarray is constructed by using the spatial smoothing technique. Then, we estimate the sparse spatial spectrum by minimizing the difference between the spatially smoothed covariance matrix and the sparsely reconstructed covariance matrix. Since the number of the sources is usually *a priori* unknown, we also devise a modified sliding window scheme for source enumeration. Finally, the power estimation is enhanced through a least squares problem.

A. Covariance Matrix Construction by Spatial Smoothing

Generally, the DOFs of source estimation problem are limited by the number of sensors. To exploit the increased number of DOFs provided by coprime array, we first construct a covariance matrix corresponding to the virtual uniform linear subarray from the sample covariance matrix corresponding to coprime array.

The sample covariance matrix $\hat{\mathbf{R}}$ can be vectorized as

$$\mathbf{z} \triangleq \operatorname{vec}(\hat{\mathbf{R}}) = \mathbf{B}\mathbf{p} + \sigma_n^2 \mathbf{i}, \tag{9}$$

where $\mathbf{B} = [\mathbf{a}^*(\theta_1) \otimes \mathbf{a}(\theta_1), \mathbf{a}^*(\theta_2) \otimes \mathbf{a}(\theta_2), \cdots, \mathbf{a}^*(\theta_K) \otimes \mathbf{a}(\theta_K)] \in \mathbb{C}^{(2M+N-1)^2 \times K}$, $\mathbf{p} = \operatorname{diag}(\mathbf{P}) = [\sigma_1^2, \sigma_2^2, \cdots, \sigma_K^2]^T$, and $\mathbf{i} = \operatorname{vec}(\mathbf{I})$. The vector $\mathbf{z}$ can be regarded as an equivalent received signal of the virtual array with the corresponding steering matrix $\mathbf{B}$. In this interpretation, $\mathbf{z}$ behaves like a single snapshot, and the rank of the covariance matrix calculated from the equivalent virtual array received signal $\mathbf{z}$ is one. Therefore, the DOAs can hardly be identified from $\mathbf{z}$ when there are multiple incident sources.

To address this problem, the spatial smoothing technique [35] is a good candidate. Specifically, since the spatial smoothing technique requires a consecutive array geometry, we first construct a matrix $\mathbf{B}_1 \in \mathbb{C}^{(2MN+1)\times K}$ by removing the repeated rows in the matrix $\mathbf{B}$ and sorting the remaining rows, so that the rows in $\mathbf{B}_1$ are identical to a consecutive virtual ULA with $2MN+1$ nominal sensors located from $-MNd$ to MNd. The new vector $\mathbf{z}_1$ can then be expressed as

$$\mathbf{z}_1 = \mathbf{B}_1 \mathbf{p} + \sigma_n^2 \mathbf{i}_1, \tag{10}$$

where $\mathbf{i}_1 \in \mathbb{R}^{2MN+1}$ denotes a zero vector except that there is a unit element at the $(MN+1)$-th position.

After the new vector $\mathbf{z}_1$ is derived, the consecutive virtual ULA can be divided into $MN+1$ overlapping subarrays with $MN+1$ nominal sensors for each subarray, where the i-th subarray has the sensors located at $\{(-i+1+n)d, n = 0, 1, \cdots, MN\}$. The equivalent received signal vector of the i-th virtual uniform linear subarray can be denoted as

$$\mathbf{z}_{1i} = \mathbf{B}_{1i}\mathbf{p} + \sigma_n^2 \mathbf{i}_{1i}, \tag{11}$$

where $\mathbf{B}_{1i} \in \mathbb{C}^{(MN+1)\times K}$ denotes the steering matrix corresponding to the $(MN+2-i)$-th through the $(2MN+2-i)$-th rows of $\mathbf{B}_1$, and $\mathbf{i}_{1i} \in \mathbb{R}^{MN+1}$ denotes a zero vector except that there is a unit element at the i-th position. Calculating the correlation statistics of each subarray received signal $\mathbf{z}_{1i}$ yields the following rank-one covariance matrix

$$\mathbf{R}_i = \mathbf{z}_{1i}\mathbf{z}_{1i}^H. \tag{12}$$

The spatially smoothed covariance matrix can then be obtained by averaging $\mathbf{R}_i$ over the $MN+1$ subarrays as

$$\mathbf{R}_s = \frac{1}{MN+1}\sum_{i=1}^{MN+1} \mathbf{R}_i, \tag{13}$$

which is now full rank, and enables us to identify up to MN sources by using only $2M+N-1$ physical sensors. As a basic step, the derived spatially smoothed covariance matrix $\mathbf{R}_s$ will be applied in the following DOA estimation problem for increasing the number of DOFs.

B. DOA Estimation Based on Sparse Reconstruction

In this subsection, we perform DOA estimation based on the idea of sparse reconstruction. Specifically, we replace K in the definition of the theoretical covariance matrix $\mathbf{R}$ (6) by a much larger integer $\bar{K}$, which denotes the number of potential sources in a predefined spatial grid. The corresponding sparse covariance matrix can be written as

$$\bar{\mathbf{R}} = \sum_{k=1}^{\bar{K}} \bar{\sigma}_k^2 \mathbf{a}(\bar{\theta}_k)\mathbf{a}^H(\bar{\theta}_k) + \sigma_n^2\mathbf{I} = \bar{\mathbf{A}}\bar{\mathbf{P}}\bar{\mathbf{A}}^H + \sigma_n^2\mathbf{I}, \tag{14}$$

where the diagonal matrix $\bar{\mathbf{P}} = \mathrm{diag}\left(\left[\bar{\sigma}_1^2, \bar{\sigma}_2^2, \cdots, \bar{\sigma}_{\bar{K}}^2\right]^T\right)$ consists of the power of $\bar{K}$ potential sources with DOAs $\bar{\boldsymbol{\theta}} = \left[\bar{\theta}_1, \bar{\theta}_2, \cdots, \bar{\theta}_{\bar{K}}\right]^T$, and $\bar{\mathbf{A}} = \left[\mathbf{a}(\bar{\theta}_1), \mathbf{a}(\bar{\theta}_2), \cdots, \mathbf{a}(\bar{\theta}_{\bar{K}})\right] \in \mathbb{C}^{(2M+N-1)\times\bar{K}}$ denotes the corresponding steering matrix. The diagonal of $\bar{\mathbf{P}}$ is sparse, which means only a few non-zero entries appear on the spatial grid corresponding to the actual source directions.

The basic idea of covariance matrix sparse reconstruction is to minimize the difference between the sample covariance matrix $\hat{\mathbf{R}}$ and the reconstructed covariance matrix $\bar{\mathbf{R}}$ under the sparse constraint. Note that this approach has been successfully applied in adaptive beamforming [36], where near optimal output signal-to-interference-plus-noise ratio (SINR) performance can be achieved. Nevertheless, when the approach is incorporated into DOA estimation, two major challenges will be encountered. First, the noise power approximated by the minimum eigenvalue of the sample covariance matrix $\hat{\mathbf{R}}$ in [36] is invalid when the number of sources is larger than the number of sensors. Second, the approximate threshold presented in [36] fails to effectively eliminate all spurious peaks, which will degrade the DOA estimation performance.

To address these challenges, we consider minimizing the difference between the spatially smoothed covariance matrix $\mathbf{R}_s$ and the sparsely reconstructed covariance matrix corresponding to the virtual uniform linear subarray

$$\tilde{\mathbf{R}} = \tilde{\mathbf{A}}\bar{\mathbf{P}}\tilde{\mathbf{A}}^H + \sigma_n^2\tilde{\mathbf{I}}, \tag{15}$$

where $\tilde{\mathbf{A}} = \left[\tilde{\mathbf{a}}(\bar{\theta}_1), \tilde{\mathbf{a}}(\bar{\theta}_2), \cdots, \tilde{\mathbf{a}}(\bar{\theta}_{\bar{K}})\right] \in \mathbb{C}^{(MN+1)\times\bar{K}}$ denotes the steering matrix of the virtual uniform linear subarray with the aperture of MNd, and $\tilde{\mathbf{I}}$ denotes the $MN+1$ dimensional identity matrix. The DOA estimation problem based on sparse reconstruction can be formulated as

$$\begin{aligned}\min_{\bar{\mathbf{p}},\sigma_n^2}\ & \|\bar{\mathbf{p}}\|_0 \\ \text{subject to}\ & \left\|\mathbf{R}_s - \tilde{\mathbf{A}}\bar{\mathbf{P}}\tilde{\mathbf{A}}^H - \sigma_n^2\tilde{\mathbf{I}}\right\|_F^2 \le \zeta, \\ & \bar{\mathbf{p}} \succeq \mathbf{0},\quad \sigma_n^2 > 0,\end{aligned} \tag{16}$$

where $\bar{\mathbf{p}} = \mathrm{diag}(\bar{\mathbf{P}}) \in \mathbb{R}^{\bar{K}}$ denotes the spatial spectrum distribution on the spatial grid, ζ denotes a specified uncertainty bound of the covariance matrix fitting error, and the ℓ_0-norm denotes the number of non-zero elements in a vector. The constraint $\bar{\mathbf{p}} \succeq \mathbf{0}$ indicates that each element of $\bar{\mathbf{p}}$ is equal to or greater than zero. The idea of the formulated optimization problem (16) lies in finding the sparsest spatial spectrum $\bar{\mathbf{p}}$ and the noise power σ_n^2 such that minimizing the fitting error between the spatially smoothed covariance matrix $\mathbf{R}_s$ and the sparsely reconstructed covariance matrix $\tilde{\mathbf{R}}$. Unlike the sparsity-based methods using an equivalent received signal vector in a single snapshot manner [32], [33], here the sparse reconstruction process is formulated based on the idea of covariance matrix fitting, and the spatially smoothed covariance matrix $\mathbf{R}_s$ is calculated from multiple equivalent virtual subarray received signals as in (13).

Unfortunately, the optimization problem (16) is NP hard because of the non-convex ℓ_0-norm, which is unsolvable even with moderately sized matrix. By introducing the ℓ_1-norm relaxation, the original non-convex optimization problem (16) can be reformulated as

$$\begin{aligned}\min_{\bar{\mathbf{p}},\sigma_n^2}\ & \|\bar{\mathbf{p}}\|_1 \\ \text{subject to}\ & \left\|\mathbf{R}_s - \tilde{\mathbf{A}}\bar{\mathbf{P}}\tilde{\mathbf{A}}^H - \sigma_n^2\tilde{\mathbf{I}}\right\|_F^2 \le \zeta, \\ & \bar{\mathbf{p}} \succeq \mathbf{0},\quad \sigma_n^2 > 0,\end{aligned} \tag{17}$$

which is convex now because the ℓ_1-norm is convex. Furthermore, the above optimization problem can be expressed as a basis pursuit denoising (BPDN) problem [37]:

$$\begin{aligned}\min_{\bar{\mathbf{p}},\sigma_n^2}\ & \left\|\mathbf{R}_s - \tilde{\mathbf{A}}\bar{\mathbf{P}}\tilde{\mathbf{A}}^H - \sigma_n^2\tilde{\mathbf{I}}\right\|_F^2 + \xi\|\bar{\mathbf{p}}\|_1 \\ \text{subject to}\ & \bar{\mathbf{p}} \succeq \mathbf{0},\ \sigma_n^2 > 0,\end{aligned} \tag{18}$$

where the regularization parameter ξ balances the tradeoff between the sparsity of the spatial spectrum and the fitting error of the reconstructed covariance matrix $\tilde{\mathbf{R}}$. The BPDN optimization problem (18) is convex and can be efficiently solved. The DOA estimation $\tilde{\boldsymbol{\theta}} = \{\tilde{\theta}_1, \tilde{\theta}_2, \cdots, \tilde{\theta}_Q\}$ can be obtained by searching for the peaks in the solved sparse spatial spectrum $\bar{\mathbf{p}}$. Meanwhile, the corresponding spatial spectrum response $\bar{\mathbf{p}}(\tilde{\boldsymbol{\theta}})$ can also be obtained.

It should be pointed out that the regularization parameter ξ is not easy to choose in different scenarios, and the balance of the tradeoff will be degraded when the regularization parameter is chosen either too small or too large. In particular, with the increase of ξ, the reconstructed spatial spectrum becomes more sparse, which may lead to the missing of the peaks of true DOAs. On the contrary, when ξ is selected to be too small, the sparsity constraint becomes invalid. Existing methods usually adopt empirical values for sparse representation (see, for example [32], [33]). Hence, it is often unavoidable that spurious peaks may appear in the reconstructed sparse spatial spectrum $\bar{\mathbf{p}}$ rather than missing the true DOAs, which leads to the overestimation of the number of the sources with a high possibility, i.e., $Q > K$. To address this issue, we consider to perform source enumeration to remove the spurious peaks in the reconstructed sparse spatial spectrum instead of investigating a systematic rule for regularization parameter selection.

C. Source Enumeration via Modified Sliding Window Scheme

In this subsection, we devise a modified sliding window scheme for source enumeration, based on which the response components in $\bar{\mathbf{p}}(\tilde{\boldsymbol{\theta}})$ are classified as either signal responses or spurious responses. Specifically, we first remove the obvious spurious response components in $\bar{\mathbf{p}}(\tilde{\boldsymbol{\theta}})$ by setting a threshold $Th = \lambda_{min}/10$, where λ_{min} is chosen as the minimum eigenvalue of $\mathbf{R}_s$. Considering the fact that the $MN + 1$ dimensional spatially smoothed covariance matrix $\mathbf{R}_s$ enables DOA estimation with the number of DOFs $\mathcal{O}(MN)$, λ_{min} is capable to represent the level of noise power. If the response component in $\bar{\mathbf{p}}(\tilde{\boldsymbol{\theta}})$ is less than one order magnitude of λ_{min}, it is undoubtedly regarded as the spurious peak and can be removed directly. Next, we sort the remaining response components $\{\bar{p}_q \in \bar{\mathbf{p}}(\tilde{\boldsymbol{\theta}}), q = 1, 2, \cdots, Q'\}$ in decreasing order as

$$\bar{p}_1 \geq \bar{p}_2 \geq \cdots \geq \bar{p}_K \geq \bar{p}_{K+1} \geq \cdots \geq \bar{p}_{Q'}, \tag{19}$$

where $Q' \leq Q$ with equality when there is no response component less than the threshold in $\bar{\mathbf{p}}(\tilde{\boldsymbol{\theta}})$.

Then, the response components in (19) are used for source enumeration based on a sliding window scheme. The idea of sliding window is to calculate the energy ratio of two consecutive sliding windows as the decision variable. When the sliding window is either entirely in the signal response category $\{\bar{p}_q, q = 1, 2, \cdots, K\}$ or entirely in the spurious response category $\{\bar{p}_q, q = K + 1, K + 2, \cdots, Q'\}$, the decision variable is nearly constant since the sliding window only contains the energy of signal response components or spurious response components. Considering that there is a noticeable energy gap between different response categories, the energy contained in a sliding window is much larger than that in the next window when the window slides over the critical point. Based on this observation, the number of the sources can be estimated as

$$\hat{K} = \arg\max_{j} \frac{w_j}{w_{j+1}}, \quad \forall\ j = 1, 2, \cdots, Q' - \kappa, \tag{20}$$

where $w_j = \sum_{q=j}^{q=j+\kappa-1} \bar{p}_q$ denotes the sum of κ response components in the j-th sliding window. The sliding window scheme requires $\kappa > 1$ according to (20). If $\kappa = 1$, the estimated number of sources $\hat{K}$ will be distorted due to the possible irregular distribution of the spurious responses. On the other hand, the number of peaks in the estimated spatial spectrum is relatively limited as compared to the sliding window applications in group detection and time synchronization [38]. Hence, here we simply choose $\kappa = 2$.

Using $\kappa = 2$, the formula in (20) indicates that there should exist at least two spurious peaks in the sorted spectrum response set $\{\bar{p}_1, \bar{p}_2, \cdots, \bar{p}_{Q'}\}$, which is not always true especially in some relatively ideal scenarios, i.e., $Q' < K+2$. To address this problem, we introduce an artificial auxiliary component λ_{min} to the spectrum responses in (19) as

$$S_p = \{\bar{p}_1, \bar{p}_2, \cdots, \bar{p}_K, \bar{p}_{K+1}, \cdots, \bar{p}_{Q'}, \lambda_{min}\}. \tag{21}$$

Subsequently, we apply the sliding window process (20) to S_p and check the critical condition $\hat{K}+2 = Q'+1$, which signifies the fact that there exists only one spurious response component in S_p, i.e., the introduced artificial auxiliary component λ_{min}. When the critical condition is satisfied, we need to add another auxiliary component λ_{min} to S_p, and the corresponding spectrum response set becomes

$$S'_p = \{\bar{p}_1, \bar{p}_2, \cdots, \bar{p}_K, \bar{p}_{K+1}, \cdots, \bar{p}_{Q'}, \lambda_{min}, \lambda_{min}\}. \tag{22}$$

The spectrum response sets S_p and S'_p for the proposed modified sliding window scheme can overcome the mismatch caused by the non-ideal critical conditions $K = Q' - 1$ and $K = Q' - 2$, respectively.

After that, the sliding window process (20) can be applied to the processed spectrum response set S_p (21) or S'_p (22), from which the number of sources is estimated. According to the estimated number of sources, we regard $\hat{K}$ largest components in $\bar{\mathbf{p}}(\tilde{\boldsymbol{\theta}})$ as the signal responses, and the DOAs can be obtained from the corresponding locations of these peaks as $\hat{\boldsymbol{\theta}}_{\mathbf{p}} = \left[\hat{\theta}_{p_1}, \hat{\theta}_{p_2}, \cdots, \hat{\theta}_{p_{\hat{K}}}\right]^T$.

D. Power Estimation Enhancement

Recall that the purpose for adopting spatially smoothed covariance matrix $\mathbf{R}_s$ is to take advantage of the increased number of DOFs for DOA estimation. However, it is worth noting that $\mathbf{R}_s$ is a correlation statistics of a set of equivalent virtual subarray received signals. Hence, we prefer to use the original sample covariance matrix $\hat{\mathbf{R}}$ to improve the accuracy of power estimation. In detail, we re-estimate the source power by matching the sample covariance matrix $\hat{\mathbf{R}}$ and the theoretical covariance matrix of the extended coprime array, where the estimated DOAs $\hat{\boldsymbol{\theta}}_{\mathbf{p}}$ is incorporated as *a priori*.

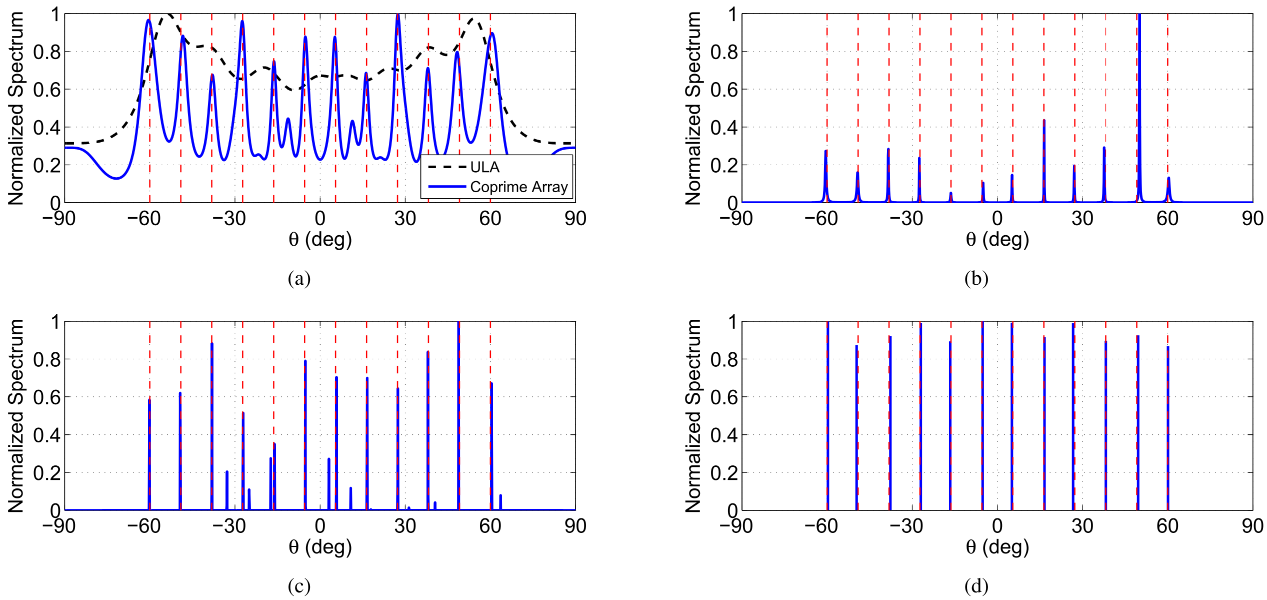


Fig. 3. Spatial spectrum comparison. Red dashed lines indicate the DOAs of incident sources. (a) Capon method in [39]. (b) Coprime MUSIC algorithm in [28]. (c) Sparse signal reconstruction algorithm in [32]. (d) Proposed source estimation algorithm.

In this regard, the optimization problem for enhancing power estimation can be formulated as

$$\min_{\bar{\mathbf{p}}(\hat{\boldsymbol{\theta}}_{\mathbf{p}})} \left\| \hat{\mathbf{R}} - \mathbf{A}(\hat{\boldsymbol{\theta}}_{\mathbf{p}})\bar{\mathbf{P}}(\hat{\boldsymbol{\theta}}_{\mathbf{p}})\mathbf{A}^H(\hat{\boldsymbol{\theta}}_{\mathbf{p}}) - \hat{\sigma}_n^2\mathbf{I} \right\|_F^2$$
$$\text{subject to } \bar{\mathbf{p}}(\hat{\boldsymbol{\theta}}_{\mathbf{p}}) \succeq \mathbf{0}, \tag{23}$$

where $\bar{\mathbf{p}}(\hat{\boldsymbol{\theta}}_{\mathbf{p}}) = \text{diag}\left(\bar{\mathbf{P}}(\hat{\boldsymbol{\theta}}_{\mathbf{p}})\right) \in \mathbb{R}^{\hat{K}}$ denotes the enhanced power estimation of the estimated DOAs $\hat{\boldsymbol{\theta}}_{\mathbf{p}}$, $\mathbf{A}(\hat{\boldsymbol{\theta}}_{\mathbf{p}}) \in \mathbb{C}^{(2M+N-1)\times\hat{K}}$ denotes the steering matrix of the extended coprime array corresponding to the estimated DOAs $\hat{\boldsymbol{\theta}}_{\mathbf{p}}$, and $\hat{\sigma}_n^2$ denotes the estimated noise power obtained from the optimization problem (18). According to [36], the optimization problem (23) belongs to an inequality-constrained least squares problem with the solution

$$\bar{\mathbf{p}}(\hat{\boldsymbol{\theta}}_{\mathbf{p}}) = \left[\mathbf{U}^H\mathbf{U}\right]^{-1}\mathbf{U}^H\mathbf{v}, \tag{24}$$

where $\mathbf{U} = \Big[\text{vec}\left(\mathbf{a}(\hat{\theta}_{p1})\mathbf{a}^H(\hat{\theta}_{p1})\right), \text{vec}\left(\mathbf{a}(\hat{\theta}_{p2})\mathbf{a}^H(\hat{\theta}_{p2})\right), \cdots, \text{vec}\left(\mathbf{a}(\hat{\theta}_{p_{\hat{K}}})\mathbf{a}^H(\hat{\theta}_{p_{\hat{K}}})\right)\Big] \in \mathbb{C}^{(2M+N-1)^2\times\hat{K}}$, and $\mathbf{v} = \text{vec}(\hat{\mathbf{R}} - \hat{\sigma}_n^2\mathbf{I}) \in \mathbb{C}^{(2M+N-1)^2}$. By matching the estimated DOAs $\hat{\boldsymbol{\theta}}_{\mathbf{p}}$ with the corresponding enhanced power estimation $\bar{\mathbf{p}}(\hat{\boldsymbol{\theta}}_{\mathbf{p}})$, we can obtain an approximate solution to the original optimization problem (16) as

$$\bar{\mathbf{p}}(\theta) = \begin{cases} \bar{\mathbf{p}}(\hat{\boldsymbol{\theta}}_p), & \theta \in \hat{\boldsymbol{\theta}}_{\mathbf{p}} \\ 0, & \theta \notin \hat{\boldsymbol{\theta}}_{\mathbf{p}}. \end{cases} \tag{25}$$

Intuitively, only $\hat{K}$ elements in $\bar{\mathbf{p}}$ corresponding to the estimated DOAs $\hat{\boldsymbol{\theta}}_{\mathbf{p}}$ are non-zero.

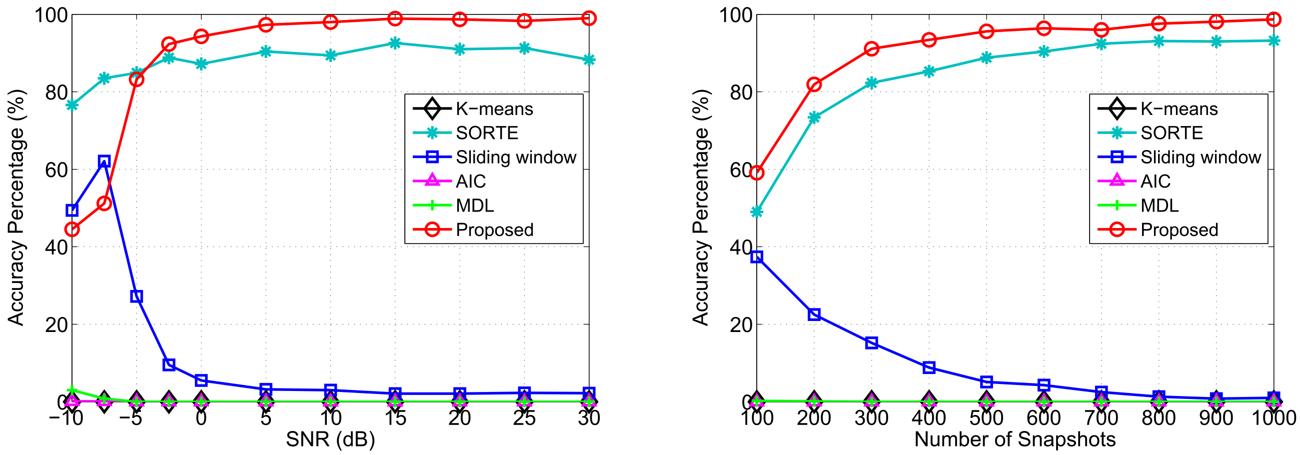
The proposed sparse reconstruction-based source estimation algorithm summarized in Table I mainly enjoys three advantages as follows. First, compared with previous estimation algorithms using ULA, the innovative coprime array geometry enables to detect more sources than sensors. Second, the number of sources can be determined through a modified sliding window scheme, from which the spurious peaks in the reconstructed sparse spatial spectrum can be effectively removed. Third, the accuracy of power estimation can be improved by solving a least squares problem. The computational complexity of the proposed source estimation algorithm is $\mathcal{O}\left(\bar{K}^2(MN+1)\right)$, which is mainly dominated by solving the optimization problem (18). Compared with the sparse signal reconstruction algorithm for coprime array in [32] with the complexity $\mathcal{O}\left(2\bar{K}(M+N-1)^2\right)$, the complexity of the proposed source estimation algorithm is slightly larger.

IV. Simulation Results

In our simulations, the extended coprime array consists of a pair of sparse ULAs with $2M = 2 \times 3 = 6$ and $N = 5$ omni-directional sensors, respectively. The array actually consists of $2M + N - 1 = 10$ sensors located at $[0, 3, 5, 6, 9, 10, 12, 15, 20, 25]d$. The additive noise is modeled as a zero-mean white Gaussian random process. It is assumed that there are $K = 12$ distinct incident source signals uniformly distributed from the directions $-60°$ to $60°$. Obviously, there are more sources than sensors. The spatial grid is uniform with $0.1°$ sampling interval within $[-90°, 90°]$. The regularization parameter ξ is empirically chosen to 0.25 for the optimization problem (18) as in [32]. For each simulation scenario, $L = 1,000$ Monte-Carlo trials are performed.

A. Spectrum Characteristic

In Fig. 3, we plot the normalized spatial spectra of the coprime Multiple Signal Classification (MUSIC) algorithm [28], the sparse signal reconstruction algorithm [32], and the proposed source estimation algorithm. For an instructive comparison, we also plot the normalized Capon spatial spectrum [39] with a fully populated ULA consisting of the same ten sensors as the coprime array. Here, the simulation



(a)

(b)

Fig. 4. Enumeration accuracy comparison. (a) Accuracy percentage versus SNR. (b) Accuracy percentage versus the number of snapshots.

TABLE I
SPARSE RECONSTRUCTION-BASED SOURCE ESTIMATION ALGORITHM USING COPRIME ARRAY

step 1: Construct the spatially smoothed covariance matrix $\mathbf{R}_s$ (13).
step 2: Solve convex optimization problem (18) for DOA estimation.
step 3: Apply modified sliding window scheme for source enumeration.
step 4: Re-estimate the power for the estimated sources through (24).
step 5: Match the re-estimated power with the estimated DOAs (25).

parameters are set to be SNR $= 0$ dB and $T = 500$, respectively.

From Fig. 3(a), we can see that the Capon spatial spectrum of ULA is unable to identify all of the sources since the available DOFs are nine. In contrast, all spatial spectra of coprime array can identify the twelve signal sources successfully. Therefore, the effectiveness of coprime array for increasing the number of DOFs is verified. Although there are more than twelve peaks in the Capon spatial spectrum using coprime array as Fig. 3(a) shows, it is evident that these peaks are not as sharp as those displayed in the other spatial spectra. Moreover, we note that most peak responses in the normalized coprime MUSIC spatial pseudo-spectrum are underestimated as Fig. 3(b) shows, and there exist some irregular spurious peaks around the signal response peaks for the spatial spectrum using the sparse signal reconstruction algorithm as Fig. 3(c) shows. In contrast, from Fig. 3(d) we can see that the normalized spatial spectrum response for each source is close to one, and there is no spurious peak. The spatial spectrum comparison shows that the proposed source estimation algorithm can decrease the spectral leakage and achieve better spectrum characteristic.

B. Source Enumeration Accuracy

In Fig. 4, we compare the source enumeration performance of the proposed modified sliding window scheme with the K-means classification method [40], the second order statistic of the eigenvalues (SORTE) method [41], the sliding window scheme [42], the Akaike Information Criterion (AIC), and the Minimum Description Length (MDL) criterion [43]. The enumeration accuracy percentage is taken as the performance evaluation measure, which calculates the percentage of correct source enumeration among Monte-Carlo trials. Considering that we have formulated the increased DOFs case, the sample covariance matrix is unable to perform source enumeration due to its limited number of eigenvalues. Instead, we use the eigenvalues of the spatially smoothed covariance matrix $\mathbf{R}_s$ to perform K-means classification method, SORTE method, AIC, and MDL criterion. When performing the K-means classification method, we set two clusters corresponding to the signal component and the spurious component, respectively. The sliding window scheme in [42] is a preliminary version of the modified sliding window scheme in this paper. When the accuracy percentage of each algorithm is compared in terms of SNR, the number of snapshots is fixed as $T = 500$, whereas when the accuracy percentage is performed in terms of the number of snapshots, the SNR in each sensor is fixed at 0 dB.

It can be seen from Fig. 4(a) that the proposed modified sliding window scheme achieves a higher enumeration accuracy than the others especially when SNR is larger than -5 dB, and the enumeration accuracy can reach up to 97% when SNR is larger than 5 dB. In contrast, the sliding window scheme in [42] suffers from performance degradation since the precondition $Q' \geq K + 2$ is no longer satisfied with the increase of SNR. Although the enumeration performance of the SORTE method is relatively stable, it can only guarantee around 90% enumeration accuracy when SNR is larger than 0 dB. The K-means classification method fails to classify the signal component and the spurious component, since it fails to generate correct cluster centroids for clustering according to the eigenvalues of the spatially smoothed covariance matrix. Meanwhile, both AIC and MDL criteria also fail to perform accurate source enumeration according to the eigenvalues of the spatially smoothed covariance matrix. In Fig. 4(b), the proposed modified sliding window scheme also shows the best enumeration performance against the number of snapshots.

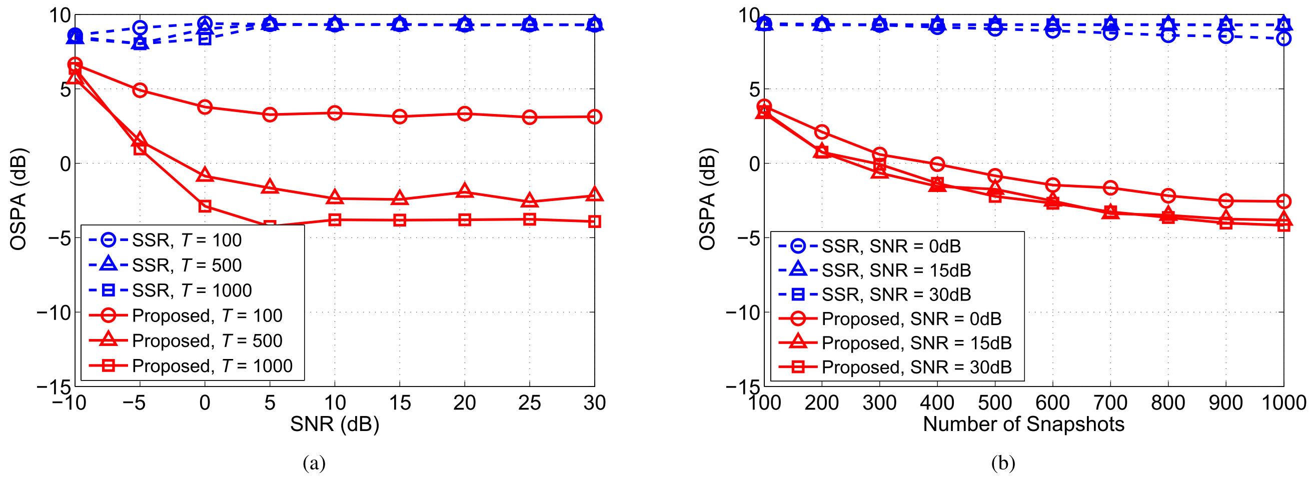


Fig. 5. DOA estimation performance comparison. (a) OSPA versus SNR. (b) OSPA versus the number of snapshots.

C. DOA Estimation Performance

In the following, we compare the DOA estimation performance of the proposed algorithm with the sparse signal reconstruction algorithm [32]. Considering the fact that $\hat{\boldsymbol{\theta}}_{\mathbf{p}}$ includes multiple estimated DOAs, the ready-made root-mean-square error (RMSE) metric is no longer valid when $\hat{K} \neq K$. In order to break this limit, we introduce a modified Optimal Sub-Pattern Assignment (OSPA) metric usually adopted in multi-target tracking [44], to evaluate the DOA estimation performance with undetermined number of the sources.

Denote $g^{(\phi)}(\hat{\theta}_p - \theta) = \min\left(\phi, |\hat{\theta}_p - \theta|\right)$ for $\hat{\theta}_p \in \hat{\boldsymbol{\theta}}_{\mathbf{p}}$, $\theta \in \boldsymbol{\theta}$, and $\boldsymbol{\Psi}_k$ denotes the k permutations of the set $\{1, 2, \cdots, k\}$. For $\hat{\boldsymbol{\theta}}_{\mathbf{p}} = \{\hat{\theta}_{p_1}, \hat{\theta}_{p_2}, \cdots, \hat{\theta}_{p_{\hat{K}}}\}$ and $\boldsymbol{\theta} = \{\theta_1, \theta_2, \cdots, \theta_K\}$, if $\hat{K} \leq K$, the OSPA of estimated DOAs $\hat{\boldsymbol{\theta}}_{\mathbf{p}}$ can be defined as

$$\mathrm{OSPA}^{(\phi)}_{\mathrm{DOA}}(\hat{\boldsymbol{\theta}}_{\mathbf{p}}, \boldsymbol{\theta}) = \sqrt{\frac{1}{LK}\sum_{l=1}^{L}\left(\min_{\psi \in \boldsymbol{\Psi}_K}\sum_{k=1}^{\hat{K}} g^{(\phi)}\left(\hat{\theta}_{p_{k,l}}, \theta_{\psi}(k)\right)^2 + \phi^2\left(K - \hat{K}\right)\right)}, \tag{26}$$

where $\hat{\theta}_{p_{k,l}}$ denotes the estimated DOA of the k-th source in the l-th Monte-Carlo trial, and ϕ denotes the cut-off parameter that assigns the relative penalty weighting to the individual DOA estimation and the enumeration bias. If $\hat{K} > K$, then $\mathrm{OSPA}^{(\phi)}_{\mathrm{DOA}}(\hat{\boldsymbol{\theta}}_{\mathbf{p}}, \boldsymbol{\theta}) = \mathrm{OSPA}^{(\phi)}_{\mathrm{DOA}}(\boldsymbol{\theta}, \hat{\boldsymbol{\theta}}_{\mathbf{p}})$. In our simulations, we choose the cut-off parameter $\phi = 10°$, which denotes the maximum spacing of the twelve uniformly distributed sources within $[-60°, 60°]$. By using the OSPA metric defined in (26), both the RMSE of each DOA estimation and the number of estimated DOAs are evaluated simultaneously.

The OSPA versus SNR is plotted in Fig. 5(a) with the number of snapshots $T = 100$, 500, and 1000, respectively. We also compare the OSPA versus the number of snapshots in Fig. 5(b) with the fixed SNR at 0 dB, 15 dB, and 30 dB, respectively. For simplicity, we use the abbreviation SSR to represent the sparse signal reconstruction algorithm [32] in the legend. We can observe from Fig. 5 that the proposed algorithm shows a significant advantage over the sparse signal reconstruction algorithm. The OSPA of the proposed algorithm decreases as the SNR increases or the number of snapshots increases. In contrast, the OSPA of the sparse signal reconstruction algorithm remains at approximately the same value, and the performance improvement is not obvious with the increase of SNR or the number of snapshots. The reason can be clearly found in Fig. 3(c), where the irregularity of spurious peaks leads to the non-ideal DOA estimation performance. Moreover, the proposed algorithm enjoys around 13 dB OSPA superiority over the sparse signal reconstruction algorithm when SNR = 30 dB and $T = 1000$.

Note that the DOAs of the sources will not always fall on the predefined spatial grids in our simulations. According to the parameter setting, only two sources are located exactly on the spatial grids. This is a typical *basis mismatch* problem, which motivated the recent off-grid DOA estimation research either using the total variation norm [29], [45] or using the atomic norm [46], [47]. Besides these, we can also consider the grid refinement method [48] to alleviate the off-grid effect and improve the accuracy of DOA estimation. In this paper, we mainly focus on the *source estimation* problem by simultaneously considering three fundamental parameters.

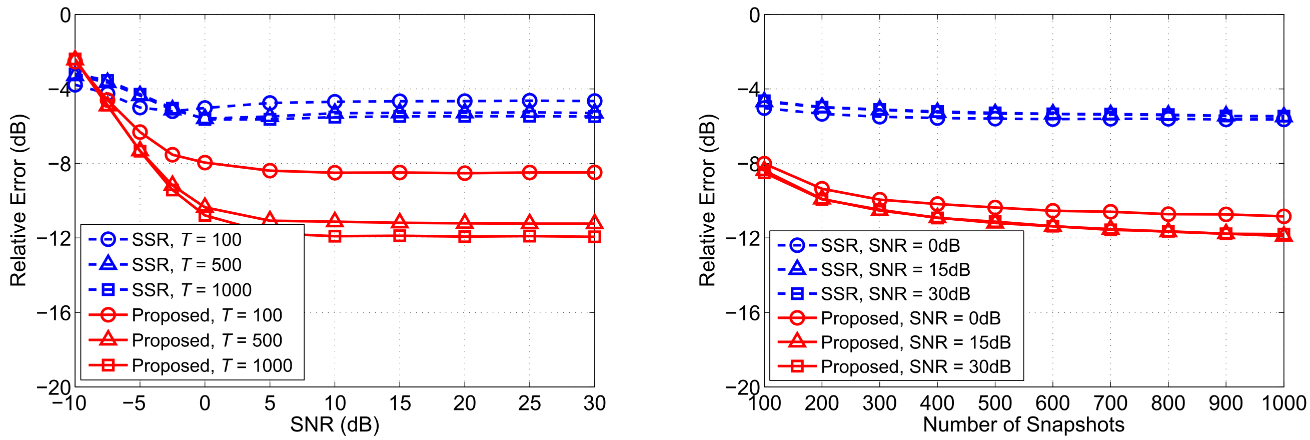
D. Power Estimation Performance

We now compare the power estimation performance of the proposed source estimation algorithm with the sparse signal reconstruction algorithm [32]. To perform a fair comparison, we assume here that the number of the sources is exactly known *a priori*, i.e., $\hat{K} = K$. We define the relative error of power estimation as

$$\frac{1}{LK}\sum_{l=1}^{L}\sum_{k=1}^{K}\frac{\left|\bar{p}_{k,l} - \sigma_k^2\right|}{\sigma_k^2}, \tag{27}$$

where $\bar{p}_{k,l}$ denotes the estimated power of the k-th source in the l-th Monte-Carlo trial.

The relative error of power estimation versus the SNR and the number of snapshots are displayed in Fig. 6(a) and Fig. 6(b), respectively. From Fig. 6(a), we can see that the relative error of power estimation for the proposed



(a) (b)

Fig. 6. Power estimation performance comparison. (a) Relative error of power estimation versus SNR. (b) Relative error of power estimation versus the number of snapshots.

algorithm is less than -10 dB when the SNR is larger than 5 dB and the number of snapshots is larger than 500. Although the power estimation performance of the proposed algorithm is slightly inferior to the sparse signal reconstruction algorithm when SNR is less than -8 dB, there exists more than 6 dB performance advantage when SNR is larger than 5 dB with $T = 1000$, which is benefited from the enhanced power estimation via the least squares problem (24). From Fig. 6(b), we can see that the proposed algorithm outperforms the sparse signal reconstruction algorithm in the whole snapshots range we considered. Instead, the sparse signal reconstruction algorithm cannot obtain more accurate power estimation by increasing the number of snapshots. That is to say, the power estimation accuracy can be effectively enhanced with the formulated least squares problem (23).

V. Conclusions

In this paper, we proposed a virtual uniform linear subarray covariance matrix sparse reconstruction-based source estimation algorithm by using coprime array. The extended coprime array makes it possible to increase the number of DOFs to $\mathcal{O}(MN)$ with $2M+N-1$ sensors, from which an optimization problem for source estimation is formulated by minimizing the difference between the spatially smoothed covariance matrix and the sparsely reconstructed covariance matrix. The source DOAs are estimated through solving the formulated optimization problem under the sparsity constraint. In order to eliminate the spurious peaks, we devise a modified sliding window scheme for source enumeration. Finally, the power estimation is enhanced through an inequality-constrained least squares problem to improve the estimation accuracy. Simulation results demonstrate the performance advantages of the proposed algorithm over other existing source estimation algorithms according to multiple evaluation metrics, such as spectrum characteristic, source enumeration accuracy, DOA estimation, and power estimation. Although the proposed source estimation algorithm simultaneously considers the three fundamental parameters and enjoys certain performance advantages, it does depend on the predefined spatial grids for sparse reconstruction. In practical applications, the actual DOAs of the signals not always fall on the predefined grids. Hence, off-grid DOA estimation problem formulation and algorithm development could be an interesting topic in future.

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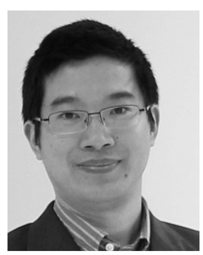

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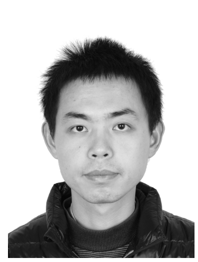

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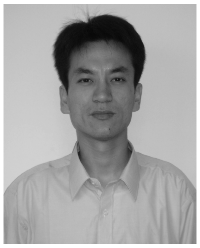

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