NOVEL ITERATIVE TECHNIQUES FOR RADAR TARGET DISCRIMINATION

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Abstract – In this paper, transmission waveforms for an *M*-target ID problem are compared and analyzed under a hypothesis testing framework. Waveforms based on mutual information between a target ensemble and the received waveforms, under time and energy constraints, have already been determined. They are applied to the *M*-target ID problem, and compared with the solution obtained by maximizing the average distance between hypotheses. A single illumination is considered first with regard to the binary and the *M*-ary target ID problem. A new iterative transmission scheme is proposed that calculates the probability of each hypothesis and updates the transmission waveform at each step. This is coupled with a sequential multi-hypotheses testing procedure and shown that for the same error rate, the proposed iterative scheme combined with an information-based waveform can reduce the number of iterations to reach a decision.

I. INTRODUCTION

There have been various attempts in the past to determine the optimum transmission waveforms for various applications. One such application is the *M*-target ID problem, where one needs to discriminate a target from other possible targets. In general, this is a hypotheses testing problem in which one needs to identify one of M (≥ 2) as the true target.

Transmit-receive optimization theory was shown as an application to the target ID problem in [5]. The solution is shown to be optimal in case of two targets, and an extension to the *M*-target problem is mentioned, but not proved or analyzed. This solution is determined by maximizing the weighted average distance between the output signals associated with the hypotheses. Another potential criterion for selecting a finite set of transmitter signals for radar is shown in [6]. This criterion maximizes the minimum divergence between hypothesis pairs being tested at the receiver. Furthermore, it is shown that the average divergence is bounded below by twice the information rate of the channel. It is claimed in [7] that the average divergence is a reasonable criterion for optimality of signal sets, and probing signals are designed for *M*-ary identification of linear channels using this criterion. Various signal design criteria and conditions for signal optimality for a communication system in a coherent Gaussian channel are derived in [4] and compared with different cases of constraints on average energy and message probabilities. In general, the regular simplex sets prove to be optimum at relatively all SNR with equal probabilities and either equal energy or bounded average power of the signals.

Information theory was applied to radar waveform design in [2] where optimal detection and estimation waveforms have been determined. It is shown that putting as much energy as possible into the mode with the largest eigenvalue may not be the best way to obtain information for identifying the target or estimating its parameters. Instead, the best estimation waveforms maximize the mutual information between the target responses (ensemble) and the received waveform.

In this paper, we look at applying the waveform determined in [2] to an *M*-target ID problem and compare with the approach given by [5] and other standard procedures using a hypothesis-testing framework. First, we consider a single illumination for binary and the *M*-ary cases. The receiver makes its decision using maximum likelihood. The difference in performance is studied. It is shown that one method maximizes the average distance between the target echoes, while the other is observed to be better at maximizing the minimum distance between echoes.

A new iterative scheme is proposed where the probabilities of the hypotheses are updated at each stage along with updating the transmit waveform. This scheme is coupled with the standard sequential multi-hypotheses testing procedure and different transmit waveforms are compared. In the sequential procedure, the figure of merit is the number of iterations to reach a decision for a fixed probability of misclassification. It will be shown that the number of iterations is considerably reduced by choosing a waveform that can extract more information about a target. In addition, it is consistent with the fact that the overall number of iterations will reduce only if you transmit the best waveform at each stage.

In Section II, we formulate the problem and describe the signal model and parameters. Section III deals with the background of the *M*-target ID problem. The solution for *M*=2 and the general form of the *M*-ary solution assumed in [5] are described. Also, the waveform that maximizes the mutual information between a Gaussian target ensemble and the received waveform is illustrated. In Section IV, we show results for the single-illumination case. We compare the error performance of different waveforms starting with binary and subsequently multiple hypotheses. Section V describes sequential testing applied to multiple hypotheses. An iterative scheme is introduced, describing the update of the probabilities and the transmit waveform at each step. This scheme is combined with the sequential multi-hypotheses testing. Results show the relative performance of different transmission techniques and the reduced number of iterations obtained by transmitting the waveform that maximizes mutual information. In Section VI, we make our conclusions.

II. PROBLEM FORMUALTION AND SIGNAL MODEL

It is assumed that we have *M* targets characterized by their impulse responses $h_i(t), i = 1,2,3,...,M$ .The impulse responses are real, time-limited and chosen as sample functions of a Gaussian random process with a specified power spectral density (PSD). The received signals are assumed to be corrupted by additive white Gaussian noise (AWGN).

Our aim is to identify the true target from among all possible targets. In a basic sense, it can be viewed as a hypothesis testing problem, where one needs to choose a hypothesis based on the received signal.

The received signal can be represented as

$$r(t) = s(t) * h(t) + n(t)$$

where $r(t)$ represents the received signal, $s(t)$ the transmit signal, $h(t)$ the impulse response of the true target, and $n(t)$ is AWGN. The discrete-time version of the above equation is used in our simulations. All impulse responses are normalized to have unit energy such that

$$\sum_{n=1}^{N} h^2(n) T_s = 1$$

where T_s is the sampling interval and *N* is the number of samples. Our problem of interest is as follows: given a known set of target impulse responses, find the waveforms $s(t)$ that maximize the probability of correct classification of the targets.

III. WAVEFORM DESIGN

The *M*-target ID problem has been considered in the literature in different ways. The transmit-receiver optimization theory in [5] was shown to be an application to the target ID problem with two targets. In the case of just two targets in AWGN, the problem transforms into maximizing the L^2 norm distance between the target echoes in signal space.

Therefore the problem becomes

$$\max_{s(t)} \int | y_1(t) - y_2(t) |^2 \, dt = \max_{s(t)} \int | y(t) |^2 \qquad (1)$$

where $y(t) = y_1(t) - y_2(t)$. The solution to this turns out to be the eigenfunction associated with the largest eigenvalue of the Fredholm integral equation of the second kind, namely

$$\lambda_{\max} s_{opt}(\tau_1) = \int_0^T s_{opt}(\tau_2) K(\tau_1, \tau_2) d\tau_2 \qquad (2)$$

$$K(\tau_1, \tau_2) = \int h^*(t - \tau_1) h(t - \tau_2) dt \qquad (3)$$

where $h(t) = h_1(t) - h_2(t)$, $\lambda_{\max}$ is the maximum eigenvalue of the kernel *K*, $s_{opt}(t)$ is the optimum transmit signal to be determined, and *K* is the kernel formed from the impulse responses as shown in (3).

Furthermore, it has been suggested that this can be extended to an *M*-target ID problem by maximizing the weighted average separation between hypotheses. The solution will then have the same form as (2) with the kernel being

$$K(\tau_1, \tau_2) = \sum_{m,n}^{\binom{N}{2}} w_{m,n} \int h_{m,n}{}^*(t - \tau_1) h_{m,n}(t - \tau_2) dt \qquad (4)$$

where $h_{m,n}(t) = h_m(t) - h_n(t)$ and $\binom{N}{2} = \dfrac{N!}{(N-2)!\ 2!}$.

The weights to be assigned to the individual pairs of hypotheses are not described in [5]. Moreover, there is no proof that (4) provides the optimal solution for an *M*-target ID problem. This particular solution will be referred to as the *eigensolution* in the later parts of this paper.

Given a Gaussian target ensemble of random impulse responses ***g(t)*** with spectral variance $\sigma_G^{\,2}(f)$, the waveforms confined to the symmetric time-interval [-*T/2,T/2*] that maximize the mutual information between the received waveform and the ensemble in additive Gaussian noise with one sided power spectral density $P_{nn}(f)$, is derived in [2]. The solution has the magnitude-squared spectrum given by

$$|S(f)|^2 = \max\left[0, A - \frac{P_{nn}(f)T}{2\sigma_G^{\,2}(f)}\right] \tag{6}$$

and A is found by solving the equation

$$E_s = \int_W \max\left[0, A - \frac{P_{nn}(f)T}{2\sigma_G^{\,2}(f)}\right] df \tag{7}$$

where E_s is the energy of the transmit signal and $P(H_i)$ is the probability of hypothesis *i*. An interesting observation is that (7) performs *waterfilling* [3] on the function $r(t) = \frac{P_{nn}(f)T}{2\sigma_G^{\,2}(f)}$. This solution will be referred to as the *waterfilling solution* in subsequent sections of this paper. Since we have finite number of known impulse responses, we do not have a Gaussian target ensemble in our case. We estimate the spectral variance of the target ensemble using

$$\sigma_G^{\,2}(f) = \sum_{i=1}^{n} |H_i(f)|^2\, P(H_i) - \left|\sum_{i=1}^{n} H_i(f) P(H_i)\right|^2 \tag{8}$$

and then apply (6) and (7). It was mentioned in [2] that the waveform whose spectrum is described by (6) is particularly useful in identifying a target or extracting information about a target, as opposed to optimum detection where one needs to maximize SNR by focusing energy into the mode corresponding to the largest eigenvalue of the target response. Subsequent sections of this paper explore the application of the above waveforms with respect to binary and *M*-ary identification.

IV. SINGLE ILLUMINATION

In this section, we explore target classification using a single active transmission. The transmit signal interacts with one of the targets and the echo is corrupted by AWGN. In general, the targets are allowed to have unequal prior probability. The receiver is assumed to perform maximum-likelihood detection.

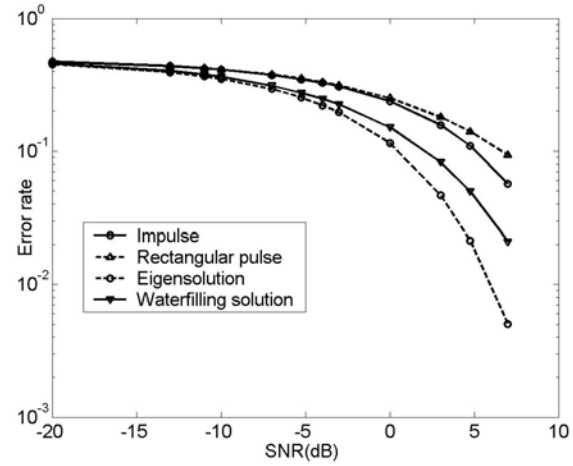

Fig.1 Error rate for two targets

A. Binary case (M=2)

When we have only two targets, it is proved that the eigensolution gives the optimal performance by maximizing SNR. This is achieved by putting maximum energy into the mode corresponding to the largest eigenvalue of the difference of target responses $h_1(t)$ and $h_2(t)$. When the targets have unequal probabilities, the detection threshold changes, while the optimal waveforms remain the same as determined for the case of equal probabilities.

The performance of this waveform is compared with a wideband impulse signal, a rectangular pulse and the waterfilling solution. Probabilities of error are determined analytically using standard methods for detection of signals in AWGN [8].

Figure 1 shows error rates in the case of two targets. The eigensolution performs the best among the waveforms considered. This is obvious from the fact that it tries to separate the two target echoes as far as possible in signal space. The waterfilling solution is also seen to perform better than the impulse and the rectangular pulse because these latter two waveforms are not intentionally matched to the target responses. The impulse outperforms the rectangular pulse because it is at least matched to the PSD from which the impulse responses were generated.

B. Multiple targets (M>2)

In the case of three targets, there is no single distance to be maximized since there are three possible distances between the hypotheses. For the *M*-ary case, prior probabilities indicate which hypotheses are most important to separate in the receive signal space. Therefore, prior probabilities affect both the transmit waveform and the detection thresholds. For unequal prior probabilities, weighting the kernels by the product of the probabilities of the two hypotheses is a reasonable approach. If one of the targets has low probability, then it is most important to separate the other hypotheses. For example, if the probabilities are P_1 = 0.4, P_2 = 0.55, and P_3 = 0.05, respectively, then we need to worry less about the distances from the third hypothesis to the other two. The weights for use in (4) would be $w_{1,2}$ = 0.22, $w_{1,3}$ = 0.02, and $w_{2,3}$ = 0.0275. Therefore, the weights reflect the relative importance of the different distances.

In [6, 7], it is shown that average divergence is a reasonable criterion for optimality, since it is bounded below by twice the information rate of the channel. The average divergence between hypotheses is given by

$$J(H) = \sum \sum P_i P_j \| y_i - y_j \|^2 \tag{8}$$

where P_i, P_j are the prior probabilities of hypotheses i and j and y_i, y_j are the target echoes with $y_i(t) = s(t) * h_i(t)$. Therefore, the solution in (4)-(5) is equivalent to maximizing average divergence in (8) when weights are chosen as a product of prior probabilities.

C. Single-Illumination Results

For our analysis, impulse responses were randomly chosen as sample functions of a Gaussian random process with flat power spectral density. For every set of impulse responses, we performed Monte-Carlo averaging over the noise. In addition, we averaged over 50 sets of impulse responses to avoid the performance being affected by the choice of a particular set of impulse responses.

The extreme case of probabilities, where one can expect to see these product weights show a significant difference in performance, is when one of the probabilities is zero or close to zero. This case is shown in Figure 2.

In Figure 3, we show results for a case with prior probabilities that are unequal, but not as drastic as in Figure 2. One interesting thing to note is that the performance of the impulse waveform approaches the optimal solutions at high SNR. This may be attributed to the fact that since the impulse responses are chosen from a process with a flat power spectrum, it is enough just to send out a wideband signal matched to the ensembles' PSD at high SNR. We are assured of not wasting energy in frequencies where there will not be a response, since the impulse responses are spread across the spectrum. Figure 4 shows that, for three equiprobable targets, the waterfilling solution approaches the eigensolution performance for high SNR.

In the case of four targets, however, there are many more distances to be maximized. For an equiprobable, 4-target situation as shown in Figure 5, the waterfilling and the

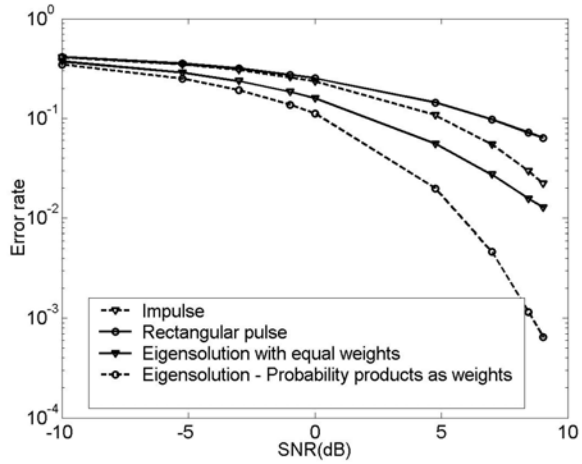

Fig.2 Error rates for P_1=0.5, P_2=0.5, P_3=0

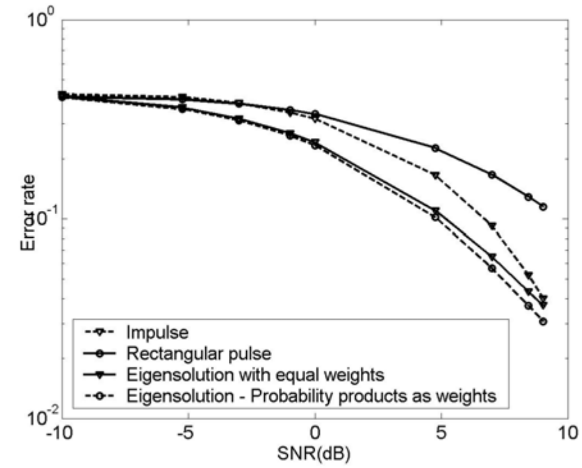

Fig.3 Error rates for P_1=0.2, P_2=0.2, P_3=0.6

eigensolution perform almost equally well for low SNR, but for higher SNR, the waterfilling solution performs better.

This illustrates the fact that as the number of hypotheses gets larger, just putting energy into the mode corresponding to the largest eigenvalue of (4) is not sufficient. The implication is that we need to spread energy into other modes as well.

The waterfilling solution seems to perform better as the number of hypotheses gets larger. As shown in (6)-(7), the waterfilling solution tends to put more energy into frequencies that have greater variance among the target frequency responses. The eigensolution though, tends to maximize the average separation between the hypotheses. Average separation, however, might be maximized by making one distance much larger than all others, resulting in many hypotheses that are not separated well at all. Since the distance in signal space is directly related to the distance in

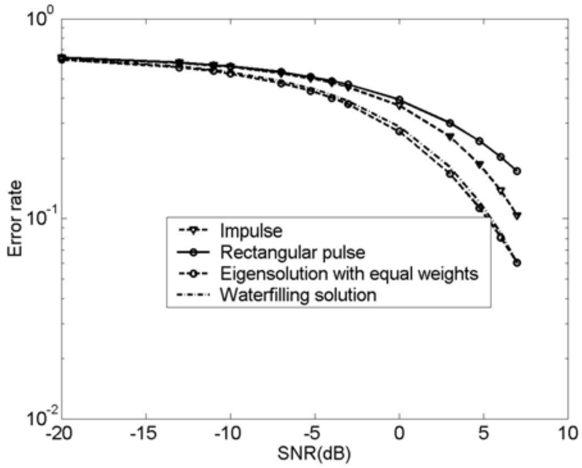

Fig.4 Error rates for the three targets equiprobable

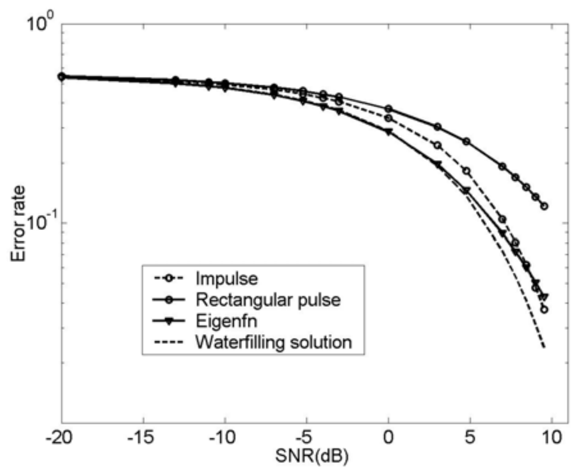

Fig.5 Error rates for 4 hypotheses

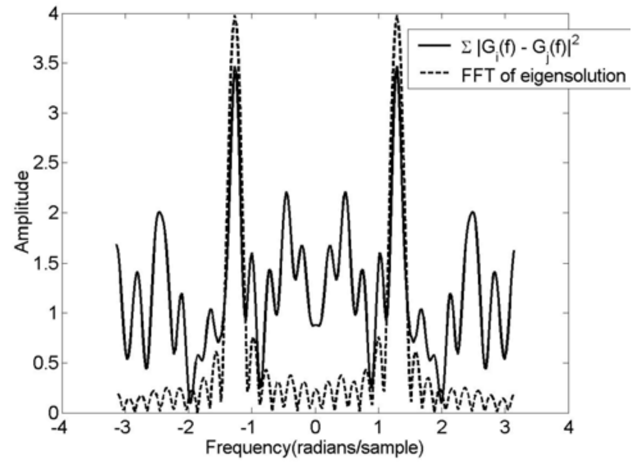

Fig6. Comparison of frequency response for eigensolution

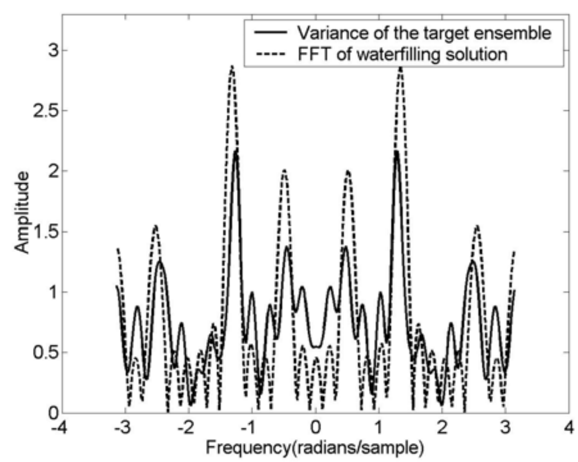

Fig.7 Comparison of frequency responses for waterfilling solution

frequency space (Parseval's Theorem), the eigensolution tends to put energy into the single frequency where the function $\sum_{i,j}^{\binom{N}{2}} |G_i(f) - G_j(f)|^2$ is the maximum, where $G_i(f)$ denotes the Fourier transform of the impulse response of i^{th} hypothesis. This concept is illustrated in Figure 6.

The waterfilling solution, however, spreads energy over multiple peaks in the ensemble's spectral variance, as long as there is enough SNR. Comparing Figures 6 and 7, we see the multiple peaks in the spectral variance function. These peaks indicate frequencies that are useful for target discrimination. At low SNR, only the largest peak is strong enough to provide useful information. At high SNR, however, there are other frequencies that can be exploited. This is achieved with the waterfilling solution.

V. ITERATIVE ADAPTIVE TRANSMISSION

Instead of making a decision after only a single step, another approach is to divide up the available energy into steps and transmit multiple waveforms. In that case, every time a signal is received, we learn something about the target. Therefore, based on all previous received waveforms, we are able to send out an improved waveform during the next transmission.

A. Bayesian Update of Probabilities

We have developed an iterative scheme, which starts out by assuming that all targets are equally probable. After each transmission and reception, the probabilities of the targets are updated using the Bayesian update rule

$$p(H_i \mid y_k) = \frac{p(y_k \mid H_i)p(H_i \mid y_{k-1})}{\sum_{j=1}^{n} p(y_k \mid H_i)p(H_i \mid y_{k-1})} \quad (9)$$

where k is the iteration number, H_i are the hypotheses, y_k is the received signal at the k^{th} iteration, and $p(H_i \mid y_k)$ is the probability of Hypothesis i after k iterations. The updated probabilities are used in determining the new waveform to be sent. In the case of fixed waveforms like impulse and rectangular waveforms, no update is performed. In case of the eigensolution, the weights, $w_{i,j}$, change with the new probabilities. With the waterfilling solution, the variance of the target ensemble is updated according to (8). This new variance is used to determine the waveform for the next transmission.

B. Sequential Hypothesis Testing

Sequential hypothesis testing is a standard procedure for testing between hypotheses by successive observations. Sequential hypothesis testing has the property that it minimizes the number of iterations to reach a decision for a fixed probability of misclassification, compared to the fixed iteration method. The iterative procedure illustrated in the previous section can be coupled with the sequential hypotheses testing procedure to observe the relative performances with respect to the number of iterations to reach a decision, for a fixed error rate.

At each iteration the likelihood ratios of all pairs of hypotheses are computed, and if the likelihood ratios for any one of the hypotheses against all others is more than a set threshold, that particular hypothesis is decided as the true one [1]. If no single hypothesis is a sufficiently clear choice, we continue to take measurements by updating the probabilities and transmission waveforms. The threshold is fixed based on the error rate set for the probability of misclassification, which is decided before the experiment begins [9].

C. Sequential-Testing Simulation Results

Sequential multi-hypotheses testing was performed with the probability of misclassification set to *0.008*. The impulse responses were randomly chosen from a Gaussian random process with a flat PSD for M = 4. The different transmit schemes were implemented assuming that the targets were equally likely. The impulse and the rectangular pulse were transmitted without any update. For both the eigensolution and the waterfilling solution, the probabilities and waveforms were updated after each transmission.

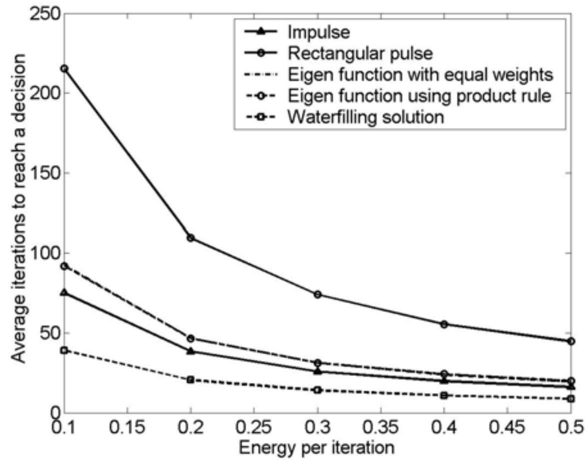

Fig.8 Iterations to reach a decision for a flat PSD

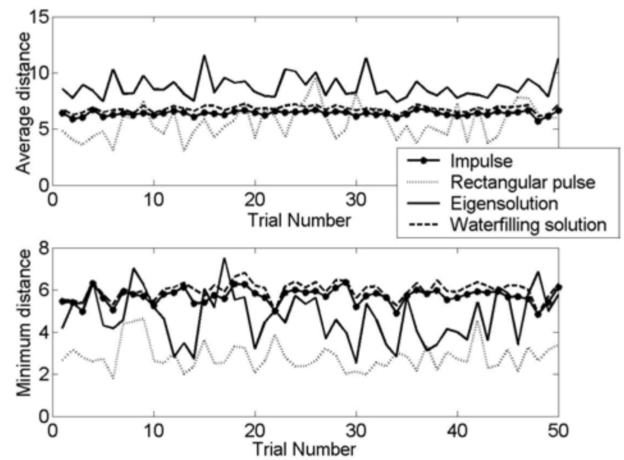

Fig.9 Distances between hypotheses for flat PSD

The waterfilling solution seems to perform the best as shown in Figure 8. It reduced the number of iterations considerably compared to the other waveforms under consideration. Assigning equal weights or product of probabilities for the eigensolution does not affect results in this case as they seem to overlap. Since the probability of error in deciding on a hypothesis depends mainly on the distance between the hypotheses, it is reasonable to investigate the minimum and the average distances between the hypotheses for different transmission waveforms. Figure 9 is consistent with the fact that the eigensolution maximizes the average separation between the hypotheses. Although the waterfilling solution does not strictly maximize the minimum distance, it does so more often than not as shown in Figure 9. This makes the waterfilling solution perform better on average than other waveforms

Next, we consider the impulse responses from a Gaussian random process with a low-pass hamming PSD, for which

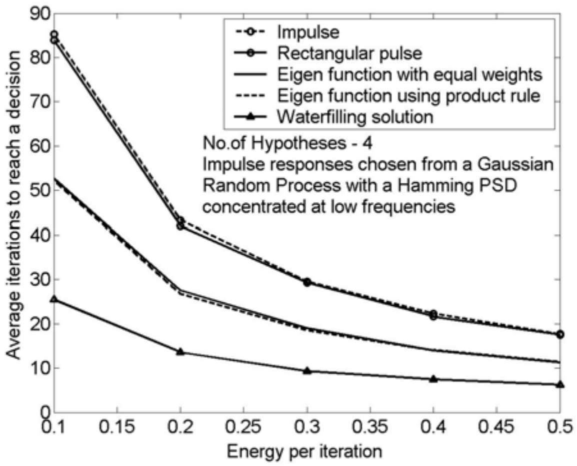

Fig.10 Iterations to reach a decision for a Hamming PSD

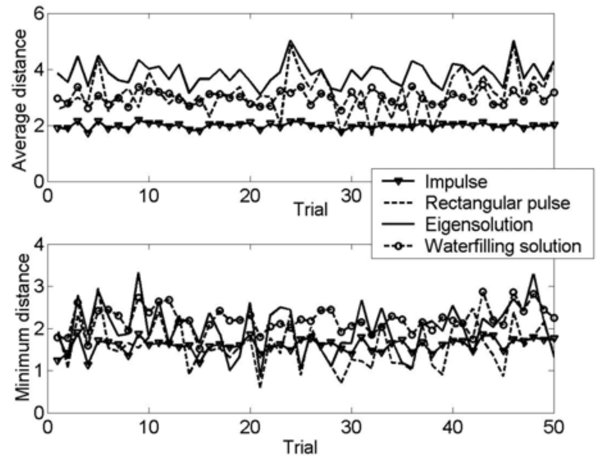

Fig.11 Distances between hypotheses for Hamming PSD

the results are shown in Figure 10. The waterfilling solution performed the best, and the wideband signal the worst. This is reflected in the minimum distances between the hypotheses in Figure 11, where the waterfilling solution maximizes the minimum distance and the impulse yields the minimum, often. Moreover, since the impulse responses contain low frequency content, transmitting wideband signal wastes energy in unwanted frequencies.

VI. CONCLUSIONS

We have analyzed various potential transmission waveforms for an *M*-target ID problem. The waveform that optimizes mutual information between a target ensemble and the received waveform was applied to this problem. The M=3 and M=4 cases were simulated, and it was seen that the waterfilling solution performs well as the number of hypotheses increase, especially at high SNR. An iterative procedure, which calculates the probabilities of targets at each step and updates the transmitted waveform, was introduced. This iterative procedure was coupled with sequential multi-hypotheses testing. It was shown that the waveform that optimizes mutual information is the best for an *M*-target ID problem for $M > 2$.

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