Channel Probability Ensemble Update for Multiplatform Radar Systems

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***Abstract*—Cognitive radar (CR) is a recently proposed concept that depicts the radar channel in a probabilistic manner. In a multiplatform or networked radar system, some parameters or dimensions of interest are visible (i.e., resolvable) to one radar and not to others depending on the geometry of the scenario. For a radar with new measurements, Bayesian methods to update the cell ensemble probabilities in the non-visible parameters are needed. Here, we show how the overall probabilistic understanding of the channel can be updated despite the fact that some cells are non-visible or "ambiguous". Unfortunately, the number of calculations needed to accomplish a full update is exponentially related to the number of cells. As such, we also introduce a technique that reduces the calculations immensely. Finally, we apply both update techniques to a two-platform radar system trying to form a two-dimensional probability ensemble of the channel.**

***Index Terms*—Multiplatform Cognitive Radar, Probability ensemble update procedure**

I. INTRODUCTION

Classical radars usually form a map of the parameters of interest using the current measurements without utilizing prior measurements or probabilities. Two of the unique features of cognitive radar (CR) are that it forms a probabilistic map of the radar channel that it is trying to depict [1-3] and that it carries this probabilistic understanding and history over many observations. Thus instead of making a map of amplitudes per resolution cell as in traditional radars, CR forms a probability map or ensemble, where each resolution cell contains a probability of a target being present. Moreover, CR incorporates prior knowledge, i.e., initial probabilities and prior measurements, in forming the current channel understanding. A decision can be made as to whether a target is present in a cell when a particular probability threshold is met. It is well known that sequential hypothesis testing (SHT) can improve probability of false alarm (P_{fa}) and/or probability of detection using multiple measurements or it can reduce the number of measurements while keeping these probabilities constant [4]. Indeed, SHT and initial prior probabilities (via the Bayesian framework) were incorporated in [2-3] to form a CR platform. Clearly, appropriate probability updating is critical in the operation of a closed-loop radar system. In the results section of the paper, we will illustrate by a simple example the benefits of incorporating prior knowledge via probability updating. The cost of updating the probabilities, however, depends on the scenario of interest. This paper deals with various scenarios and focuses on a difficult case; the case where a great number of cells are considered "ambiguous". The use of the word "ambiguous" in this context will be explained further in the next paragraph.

In [5], we considered a one-platform CR for an integrated search and track application. The CR in [5] used the probabilistic understanding of the channel to form adaptive matched illumination via mutual information optimization [6-8]. For a multiplatform system, multiple radars may cooperate to form a multidimensional map of the channel and use the probabilistic understanding to cooperatively illuminate the channel. In such a networked system, a particular radar may not be able to "see" or measure all parameters of interest due to its geometry. For example, consider a radar that is able to measure parameter x but not parameter y, and another radar that is able to measure parameter y but not parameter x. For convenience, we assume that the number of resolution cells in each dimension is $M = M_x = M_y$. The two radars can network to form a two-dimensional probabilistic understanding of the x, y parameter space. The radar that can measure x is totally blind to variations in y, i.e., for every x-resolvable cell, there are M y cells that are indistinguishable or "ambiguous" albeit not necessarily in the traditional sense such as in range/Doppler ambiguities for a PRF radar. Similarly, for the other radar, for every y-resolvable cell, there are M x cells that are ambiguous. Thus, the cooperative system needs a way to update the probability ensemble after each radar illuminates the channel. This paper reports a method to update the probabilities for this scenario. Depending on the number of cells that are ambiguous, the full procedure may be computationally prohibitive. Consequently, we also present a technique that greatly reduces the number of calculations required to make the probability updates.

II. SCENARIO MODELING

We consider two sensors shown in Fig. 1(a). Sensor A can measure a parameter θ_x and not θ_y and vice versa for Sensor B. For example, each sensor might be used to measure angular space. As seen in Fig. 1(a), the channel may be described by a two-dimensional map, where each cell is described by its θ_x, θ_y coordinate vector. The use of only two parameters here provides a proof-of-concept test case for the probability ensemble update procedures discussed in this paper. These techniques apply to many different multidimensional scenarios where multiple radar platforms observe multiple target position and velocity components. In our simple model, since the two sensors are able to measure two parameters, albeit separately, the overall probability ensemble is clearly two-dimensional. Since there are M θ_x cells and M θ_y cells, there are M^2 cells in the two-dimensional parameter space. For Sensor A, note under each θ_x-cell, there are M θ_y cells that are ambiguous in the sense that Sensor A cannot resolve them. Similarly, for Sensor B, under each θ_y-cell, there are M cells that are ambiguous.

The probabilistic description of the channel in this case is a list of M^2 probabilities. Each probability value quantifies the chance that a target is present in one of the M^2 cells, and the goal of the update procedure is to accurately update these probabilities as new data are received. We call these probabilities the *cell probabilities*. For any individual cell, there are two possible states: target present or absent. We assume the presence/absence of a target in one cell is independent of the presence/absence of targets in other cells. Therefore, there are 2^M possible permutations of the overall target environment. Each permutation is a unique combination of target presence or absence across the resolution cells, and each permutation is itself characterized by a probability of being true.

To illustrate how multiple radars could cooperate to update a multidimensional ensemble, first we address how a single radar measuring a parameter θ with M cells as shown in Fig. 1(b) would update its cell probabilities with received measurements. For this scenario, let the one-dimensional vector of prior cell probabilities be given by

$$\mathbf{P}_0 = [\ P_{M,0} \quad \dots \quad P_{m,0} \quad \dots \quad P_{2,0} \quad P_{1,0}\ ]$$

where $P_{m,0}$ is the initial prior for the m^{th} cell. Consider a sensor that produces an N-element measurement vector with each data collection. These measurements might be N slow-time measurements for the purpose of measuring Doppler, N spatial measurements for measuring angle, or any other combination of measurements. Let $\mathbf{s}_m$ be the signal produced at the radar if a target is present in the m^{th} resolution cell, and let the measurement indices be denoted by $n = 0, 1, \dots, N-1$. For a radar system, any target parameter can be described as a frequency (e.g., Doppler, spatial, or range), so we let the frequency produced by the presence of a target in the m^{th} cell be denoted by f_m. When these cell frequencies are the same, the cells cannot be resolved and are considered ambiguous. When the frequencies differ, it may be possible to resolve the cells. The signal produced by a target in the m^{th} cell is proportional to a normalized steering vector given by

$$\mathbf{s}_m = \frac{1}{\sqrt{N}} \exp\left(j2\pi f_m [0 \cdots n \cdots N-1]^T\right). \qquad (1)$$

If we are to perform a probability update, it is necessary to assume a probability model for the measurements. For convenience, let the targets be deterministic, i.e., known amplitude and phase and let the noise be additive white Gaussian.

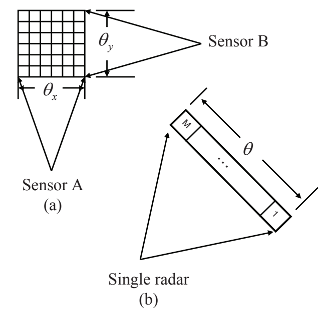

Fig. 1. (a) Two sensors forming the probability map: Parameter θ_y is ambiguous to Sensor A and parameter θ_x is ambiguous to Sensor B (b) Single radar updating a one-dimensional probability ensemble

Obviously other scenarios may produce measurements with different *pdf*s. The probability update procedures discussed in this paper are applicable to these cases as well with proper substitution of the measurement *pdf*. Recalling that there are 2^M possible permutations of the overall target environment, the environment can be described by a multiple hypotheses framework. The hypotheses in this framework are given by

$$\begin{aligned} &H_0 : \mathbf{z} = \mathbf{n} \\ &H_1 : \mathbf{z} = \mathbf{s}_1 + \mathbf{n} \\ &H_2 : \mathbf{z} = \mathbf{s}_2 + \mathbf{n} \\ &H_3 : \mathbf{z} = \mathbf{s}_1 + \mathbf{s}_2 + \mathbf{n} \\ &H_4 : \mathbf{z} = \mathbf{s}_3 + \mathbf{n} \\ &\vdots \\ &H_{2^M-1} : \mathbf{z} = \mathbf{s}_1 + \mathbf{s}_2 + \cdots + \mathbf{s}_M + \mathbf{n}. \end{aligned} \qquad (2)$$

Each hypothesis H_i may be thought of as a "joint" hypothesis corresponding to a unique permutation of target presence/absence in the individual cells. For notational convenience, we convert the joint hypothesis subscript i to its binary representation of $i = 0\cdots 00, 0\cdots 01, 0\cdots 10, 0\cdots 11, \dots, 1\cdots 11$, where a 0 corresponds to target absent and a 1 corresponds to target present in a cell. For example, consider the eleventh hypothesis in a five-cell scenario. If we let S_i correspond to the target signal produced by the i^{th} joint hypothesis, then the eleventh hypothesis would be represented as H_{01011} and the received signal contribution would be $S_{01011} = \mathbf{s}_4 + \mathbf{s}_2 + \mathbf{s}_1$.

III. Probability Updates

A. The Update Procedure for the General Case

The *pdf* for the measured data under the i^{th} joint hypothesis may now be compactly given by

$$p(\mathbf{z}|H_i) = \frac{1}{\pi^N \sigma^{2N}} \exp\left(-\frac{1}{\sigma^2}(\mathbf{z} - S_i)^H (\mathbf{z} - S_i)\right). \qquad (3)$$

The goal is to update the individual cell probabilities, but in general the radar cannot observe each cell apart from the others. Thus, we must first update the joint hypotheses using Bayes rule, which states that the posterior probability for each joint hypothesis is given by

$$P(H_i|\mathbf{z}_k) = \frac{P(H_i|\mathbf{z}_{k-1})p(\mathbf{z}_k|H_i)}{p(\mathbf{z}_k)} \quad (4)$$

where $P(H_i|\mathbf{z}_{k-1})$ is the probability of the i^{th} joint hypothesis prior to collecting the current (k^{th}) measurement. While the denominator of (4) may not readily be available, it is the same for all joint hypotheses and serves only to normalize the probabilities such that they sum to unity. Thus (4) simplifies to

$$P(H_i|\mathbf{z}_k) = \beta P(H_i|\mathbf{z}_{k-1})p(\mathbf{z}_k|H_i) \quad (5)$$

where β can be computed after evaluating (4) for all joint hypotheses.

There is still the matter of calculating the probability of joint hypothesis H_i prior to the kth measurement: $P(H_i|\mathbf{z}_{k-1})$. Recall that we have M target parameter cells each described by a probability of a target being present in that cell. The length-M probability vector that contains these probabilities prior to the kth measurement is

$$\mathbf{P}_{k-1} = [\ P_{M,k-1} \quad \ldots \quad P_{m,k-1} \quad \ldots \quad P_{1,k-1}\ ]. \quad (6)$$

Let b_1 through b_M be the individual bits of the binary representation of a joint hypothesis. For example, the eleventh joint hypothesis in the five-cell scenario described above would have $b_5 = 0$, $b_4 = 1$, $b_3 = 0$, $b_2 = 1$, $b_1 = 1$. Since target presence or absence is assumed to be independent across cells, the probability of the i^{th} joint hypothesis is

$$P(H_i|\mathbf{z}_{k-1}) = \prod_{c=1}^{M}(P_{c,k-1})^{b_c}(1 - P_{c,k-1})^{1-b_c}. \quad (7)$$

We desire to arrive at the updated cell probabilities. Once a measurement $\mathbf{z}_k$ is received, it is used to update the probabilities for all "joint" hypotheses. First, 2^M likelihoods must be evaluated as dictated by (3). Then all of the 2^M joint probabilities must be updated by (4) where (5) ensures summation to unity. The cell probabilities are obtained through the marginal probabilities of the joint hypotheses. To calculate the marginal for the m^{th} cell, we sum up the probabilities for any joint hypothesis that has a target-present state for that cell. The resulting sum is the updated probability for that cell, and the process must be done for all cells.

B. Special Case: Orthogonal Targets

If the signals produced by targets in different cells are resolvable by the radar, it can be shown that each cell can be treated independently. In other words, while the multiple hypothesis testing framework in (2) applies (since it applies to all situations in general), it is computationally more efficient to perform separate probability updates for each the M cells. If the signals due to different cells are orthogonal, then separable probability updates is equivalent to the full procedure of updating joint probabilities and then calculating the individual marginal probabilities. For any m^{th} cell, let $\mathcal{H}_{m,0}$ and $\mathcal{H}_{m,a}$ signify the target null and present hypotheses. Recall from (1) that $\mathbf{s}_m$ is the target signal that corresponds to the m cell, then the *pdf* for this cell is given by

$$\begin{aligned} p(\mathbf{z}|\mathcal{H}_{m,0}) &= \frac{1}{\pi^N\sigma^{2N}}\exp(-\frac{1}{\sigma^2}\mathbf{z}^H\mathbf{z}) \\ p(\mathbf{z}|\mathcal{H}_{m,a}) &= \frac{1}{\pi^N\sigma^{2N}}\exp\left(-\frac{1}{\sigma^2}(\mathbf{z}-\mathbf{s}_m)^H(\mathbf{z}-\mathbf{s}_m)\right). \end{aligned} \quad (8)$$

Recall that the cell probabilities at the $(k-1)^{th}$ update are described by (6). In the case where the cells are resolvable, the Bayesian rule now applies to each cell instead of target cell permutations, i.e.,

$$P(\mathcal{H}_{m,0}|\mathbf{z}_k) = \frac{P(\mathcal{H}_{m,0}|\mathbf{z}_{k-1})p(\mathbf{z}_k|H_{m,0})}{p(\mathbf{z}_k)} \quad (9)$$

$$P(\mathcal{H}_{m,a}|\mathbf{z}_k) = \frac{P(\mathcal{H}_{m,a}|\mathbf{z}_{k-1})p(\mathbf{z}_k|H_{m,a})}{p(\mathbf{z}_k)}. \quad (10)$$

Since (10) and (11) sum to one, they simplify to

$$P(\mathcal{H}_{m,0}|\mathbf{z}_k) = \alpha P(\mathcal{H}_{m,0}|\mathbf{z}_{k-1})p(\mathbf{z}_k|H_{m,0}) \quad (11)$$

$$P(\mathcal{H}_{m,a}|\mathbf{z}_k) = \alpha P(\mathcal{H}_{m,a}|\mathbf{z}_{k-1})p(\mathbf{z}_k|H_{m,a}) \quad (12)$$

where α ensures unit probability.

In summary, when the target cells are orthogonal, the approach of considering multiple hypotheses with 2^M *pdf*s and the present approach of independently considering M binary hypothesis testing are equivalent, but the latter is clearly computationally efficient.

IV. Reduced Probability update procedure

A. The Update Procedure for the General Case

When the cells are not orthogonal, i.e., not separable, the number of *pdf*s to be evaluated (3) can be very large since the total number of hypotheses to be considered is exponential in nature, i.e., 2^M. Fig. 2 shows the number of hypotheses as a function of number of cells from 4 to 32 cells. Note, in the case of 32 cells, the total number of hypotheses is astoundingly large (4.2950e+9). Fortunately, in many practical cases, the number of targets in the area of interest is usually very low. Thus, if we assume the maximum number of targets, say K, we immediately lower the number of possible joint hypotheses or permutations to consider, which is given by a summation of combinatorial functions in the form

$$C_K = \sum_{k=0}^{K}\begin{pmatrix} M \\ K \end{pmatrix}. \quad (13)$$

Fig. 2 also shows the number of hypotheses when we assume the maximum number of targets is $K = 3$. For 32 cells, the total number of hypotheses for a maximum of 3 targets is 5489.

Since we assumed the number of targets to be K or less, then the update procedure in Section III has to be modified. Recall, the reduced number of hypotheses to be considered in (3) for Bayesian updates is given by (13). To reduce the number of calculations, we will only utilize the prior joint

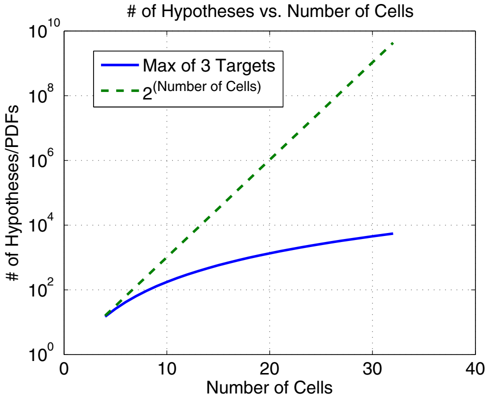

Fig. 2. Number of hypotheses versus Number of Cells

probabilities for these hypotheses, which can be evaluated by (7) via utilizing the prior marginals from (6). By utilizing the reduced number of prior joint probabilities, we should note that for a particular cell, summing up the probabilities for any joint hypothesis that has target-present state for that cell does not result in the original marginal from (6) for that cell. In effect, we ignored probability contributions from joint hypotheses corresponding to the cases where number of targets are larger than K. Once a measurement is received, (3) is evaluated and (4) is calculated, where (5) ensures unit probability. The marginal cell probabilities are calculated from the updated joint probabilities of (5), where the marginal for the m^{th} cell is derived by summing up the probabilities for any joint hypothesis that has a target-present state for that cell.

B. Special Case: Ambiguous Targets

Consider again the single radar in Fig. 1(b) where the M cells being illuminated are indistinguishable from each other. In other words, if we let $\mathbf{s}$ represent the target signal, then any cell is described by $\mathbf{s}$ if a target is present. In this case, there are only $M+1$ types of hypotheses *pdf*s to consider as opposed to 2^M hypotheses in the previous update procedure. This is because all the *pdf*s containing p number of targets, where $0 \leq p \leq M$, take the same form of

$$p(\mathbf{z}|\mathrm{H}_p) = \frac{1}{\pi^N \sigma^{2N}} \exp\left(-\frac{1}{\sigma^2}(\mathbf{z}-\mathbf{s}_p)^H(\mathbf{z}-\mathbf{s}_p)\right) \quad (14)$$

where H_p is any joint hypothesis containing p targets and $\mathbf{s}_p = p\mathbf{s}$. Thus only $M+1$ *pdf*s need to be calculated; the null hypothesis *pdf* and the M *pdf*s corresponding to $p = 1, \ldots, M$ target hypotheses.

It follows that if we assume a maximum of K targets where $K < M$, then we only need to calculate $K+1$ *pdf*s. For example, if we assume a practical case of $K = 2$, we only need to calculate 3 *pdf*s as opposed to 2^M. For instance, following on the five-cell example earlier, the hypotheses H_{01001} and H_{10001} would produce the same conditional *pdf* $p(\mathbf{z}|\mathrm{H}_2)$ since the likelihood depends only on the number of targets and not the actual target parameters. Hence, all the probabilities for the joint hypotheses with the same number of targets get scaled by the same value.

V. Observations and Results

As stated in the introduction, we will show via a simple example the benefits of probability updating by incorporating prior probability and measurements in subsection V.A. In subsection V.B, we consider how the single radar in Fig. 1(b), with multiple iterations, would utilize both update techniques described in III and IV (which we will presently term as full update procedure and the reduced update procedure respectively) when all of the M cells of the θ parameter are not separable. While there are a lot of interesting cases, we will present a few that lend us valuable insights as to how the procedures update the probabilities in the ambiguous cells. Finally in subsection V.C, we will look at how we can utilize both procedures in our scenario of interest, i.e., the collaborative environment of Fig. 1(a) where the two radars cooperatively update the probability ensemble of a two-dimensional angular space.

A. Simple Probability Updating Example

Consider a deterministic signal with amplitude A with additive white Gaussian noise such that the SNR is 0dB. To compare the performance of a classical radar to a radar that incorporates prior knowledge and measurements, consider a number of iterations r. It is known for classical radars that the probability of at least one detection in r measurements [9] is given by $P_{cd} = 1-(1-P_d)^r$ where P_d is the probability of detection for a single measurement given a fixed SNR and P_{fa}. For a 0dB SNR and $P_{fa} = 0.01$, the P_d is about 0.1 [10]. For the radar incorporating previous measurements, we will also use 0dB SNR such that we know 0.1 is the initial prior. The probability update for a target being present for each measurement is random since noise changes from iteration to iteration. We conduct a Monte Carlo simulation with 100,000 trials to find the average probability of a target present for each measurement iteration and compare that to the classical expression given above. Fig. 3 shows the comparison between the probability of at least one detection for the classical radar and the probability of detection (target being present) for a radar incorporating the prior updates over increasing measurements r. It is clear that radar system incorporating prior knowledge has a higher probability of detection over increasing iteration. The improvement in probability of detection for the closed-loop radar is a direct consequence of carrying over information from prior measurements via the probabilistic channel representation.

B. Single Radar Updating Ambiguous Cells

Recall the situation depicted by Fig. 1(b), where our specific interest is the case where the M θ cells are ambiguous. We will use the full update procedure and reduced update procedure and compare the results of a single experiment. We consider an eight-element sensor to measure the received signal. The signal to noise (SNR) is 0dB. Here, we will consider a ten-iteration update where the noise changes from measurement to measurement. We want to show and investigate the probability update progression of the M θ cells using both update

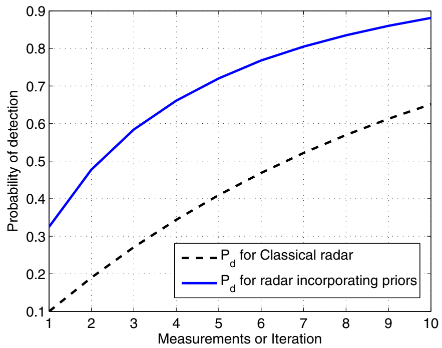

Fig. 3. Comparison between Classical radar and radar incorporating priors

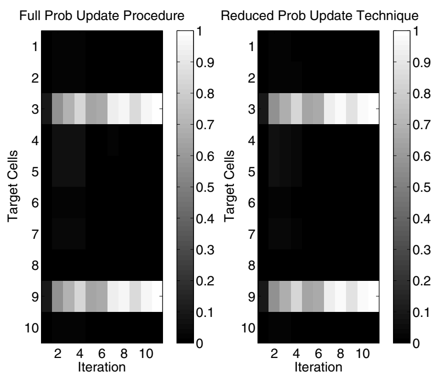

Fig. 4. Priors Agree with True Target Scenario

procedures for various situations. We are also interested in how much the reduced update procedure differs from the full update procedure and understand its shortcomings. Following are various examples.

1) Priors Agree with True Target Scenario: This is a sample experiment where priors reflect the truth scenario, i.e., with K targets, the corresponding K cells have the largest prior probabilities. In this example and the succeeding example, the two targets are placed in θ-cell #3 and θ-cell #9. Fig. 4 shows the result of ten-iteration probability update histories of a ten-cell ensemble. Left panel of Fig. 4 shows the probability updates for θ-cell #3 and θ-cell #9 are obviously the most favored at every iteration. Since the initial priors for θ-cell #3 and θ-cell #9 are equal, the updates are almost identical. For the reduced update procedure, we assumed two to be the maximum number of targets and the probability update history is shown at the right panel of Fig. 4. While there are obviously numerical differences, visually, it is difficult to see the difference between the probability update histories of the full and reduced update procedures

2) Equal Priors: In this sample experiment, the initial priors for all ten cells are equal (set at 0.01). Left panel of Fig. 5 shows the ten-iteration probability history utilizing full update procedure. The interesting fact is that probability updates for all the cells are the same when the priors are all equal. Such a result is not surprising since all the cells

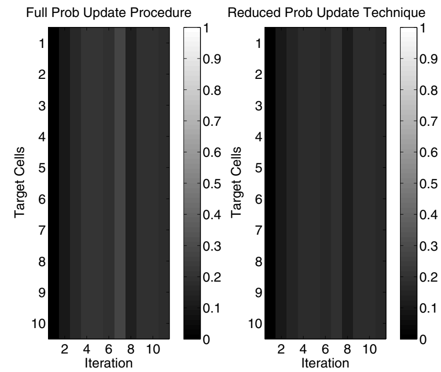

Fig. 5. Equal priors

are ambiguous. Right panel of Fig. 5 shows the ten-iteration probability history utilizing reduced update procedure. Here, visually, it is a little easier to tell the difference between the histories, i.e., the updates are not quite identical (in values) but clearly in both procedures the marginal updates for each iteration are equal.

3) Actual Number of Targets Exceeds Assumed Maximum: We have shown how the reduced procedure updates resemble the full procedure updates. The reduced update procedure for ambiguous cells is based on the assumption that that there is a maximum number of targets such that the number of joint hypotheses to be considered is greatly reduced. Now we explore a sample experiment when the maximum number of target is violated and investigate the effects of the reduced update procedure. In this example, there are three targets located at θ-cell #1, θ-cell #3, θ-cell #9 with the following initial probabilities: Pr(θ-cell #1)=0.2, Pr(θ-cell #3)=0.15, Pr(θ-cell #9)=0.1 and the rest with Pr$<$0.01. Left panel of Fig. 6 shows the probability history of the full update procedure. Note that all three cells have been updated favorably as expected. Right panel of Fig. 6 shows the probability history of the reduced update procedure which assumed a maximum of two targets. Note that in the initial stages of the update history, θ-cell #9 updates seemed promising. But since the reduced update procedure assumed a maximum of two targets only the the two cells with the highest probability were eventually favored; θ-cell #1 and θ-cell #3. Thus, this is clearly one disadvantage of the reduced update procedure; that it will only eventually favor the maximum number of targets assumed despite the presence of more targets.

C. Two-Radar Cooperative Platform

Recall Fig. 1(a), where two sensors or radars cooperate to update the two-dimensional probability ensemble despite ambiguities. The previous section showed examples on how a single radar updates cell probabilities in the ensemble that are ambiguous with multiple receptions or iterations. Instead of multiple receptions for each radar prior to an update, the eventual scenario of interest is to have each radar transmit and process measurements alternately, i.e., update the ensemble alternately. We consider a two-radar platform, with each radar

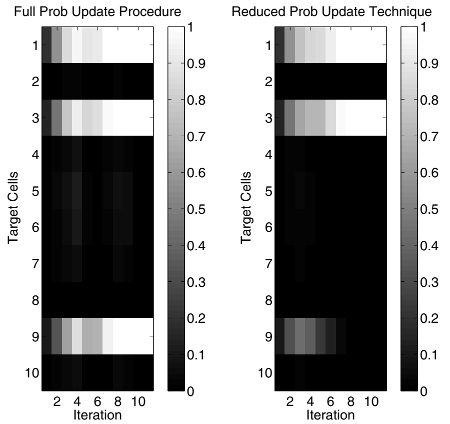

Fig. 6. Actual Number of Targets Exceeds Assumed Maximum

only able measure one parameter as in Fig. 1(a). The cooperative system is trying to map a 8-by-8 cell angular space. In this sample experiment, target signals are deterministic signal in white noise with 0 dB SNR. There are four sets of iterations, i.e., eight alternating received measurements with each radar, equipped with an 8-element sensor, receiving four measurements. There is one target and it is located in (2,2) cell. The initial priors of are randomized (0<Pr<0.1 for all cells). Left panel of Fig. 7 shows the four-set iteration probability history using the full update procedure. After iteration set #1, the probability of (2,2) cell increased from 0.0773 (which was one of the lower initial priors) to 0.4845, which became the largest at this iteration set. Other probabilities increased or decreased depending on the noise realization and initial priors. By iteration set #2, the probability in cell (2,2) increased to 0.9911. In iteration set #3, it lowered down to 0.9571. In iteration set #4, it increased to 0.9973. Right panel of Fig. 7 shows the probability history using the reduced update procedure. The probability iteration history for cell (2,2) went the following way: 0.0773, 0.4687, 0.9856, 0.8475, and 0.9892. For this particular experiment, all the updates from the reduced procedure were slightly lower compared to the full update procedure.

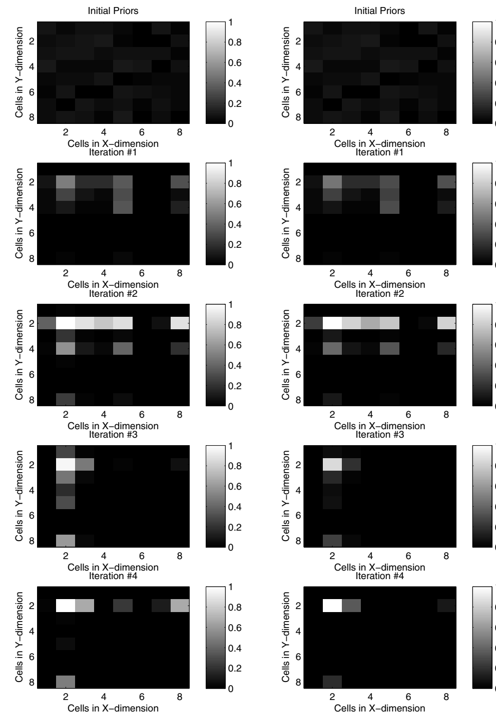

Fig. 7. 2-Radar Platform: Full and Reduced update procedure comparison

VI. Summary and Conclusion

Cognitive radar is a system concept that depicts the radar channel probabilistically. Thus, there is a need to properly update the probability ensemble. We presented the a procedure to update cell probabilities in various cases. Unfortunately, the number of calculations for the update procedure is exponentially related to the number of cells and therefore can become computationally impractical. Consequently, we presented a reduced update procedure based on constraining the number of targets. A particular application of interest is when cells are completely ambiguous. We presented examples on how a single radar would utilize the full and reduced update procedures to update the cell probabilities in the ambiguous cells with multiple iterations. Finally, we presented a cooperative two-platform radar system trying to form a two-dimensional probability ensemble using both update procedures for a sample scenario.

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